Question Bank In Mathematics Class IX (Term II)

9 AREAS OF PARALLELOGRAMS AND TRIANGLES

A. SUMMATIVE ASSESSMENT

9.1 FIGURES ON THE SAME BASE AND BETWEEN THE SAME PARALLELS

1. If two figures A and B are congruent, they must have equal areas.

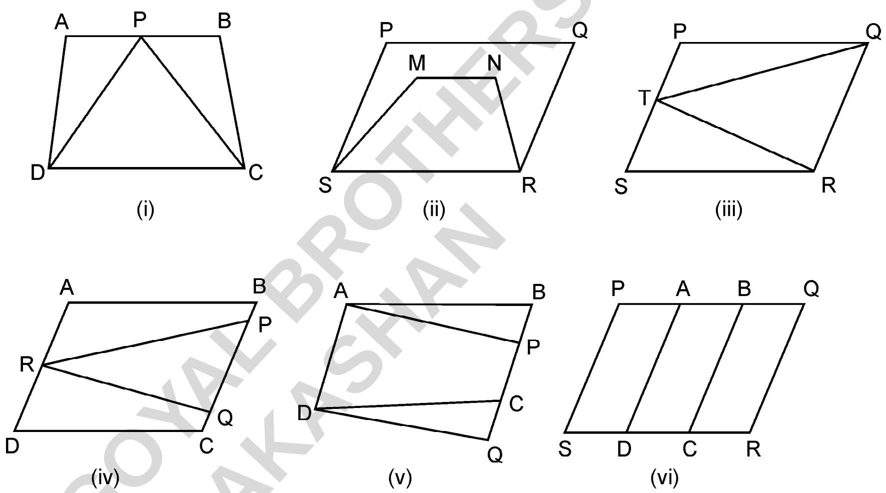
Or, if A and B are congruent figures, then ar(A) = ar(B)

2. If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then ar(T) = ar(P) + ar(Q).

3. Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

TEXTBOOK'S EXERCISE 9.1

Q.1. *Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.*



Sol. (i) Base DC, parallels DC and AB (iii) Base QR, parallels QR and PS

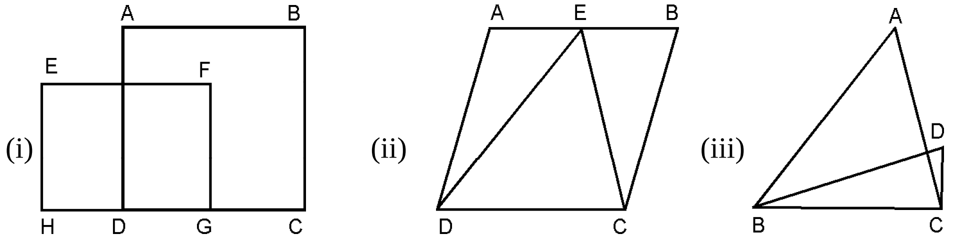
(v) Base AD, parallels AD and BQ.

PRACTICE EXERCISE 9.1A

1 Mark Questions

Choose the correct option (Q 1 – 2) :

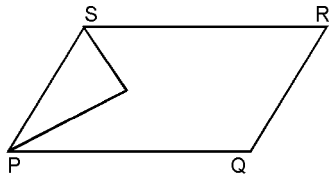
1. Which of the following figures lies on the same base and between the same parallels ? **[Imp.]**



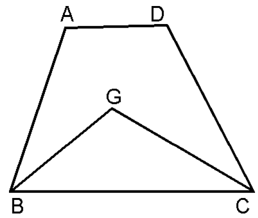
(*a*) only (i) (*b*) both (i) and (ii) (*c*) only (iii) (*d*) only (ii)

2. In which of the following figures, you find two polygons on the same base and between the same parallels?

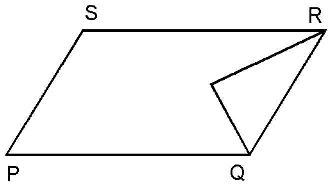
(*a*)



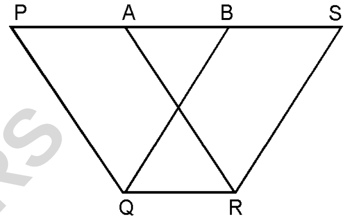
(*b*)



(*c*)



(*d*)



9.2 PARALLELOGRAMS ON THE SAME BASE AND BETWEEN THE SAME PARALLELS

1. Parallelograms on the same base and between the same parallels are equal in area.

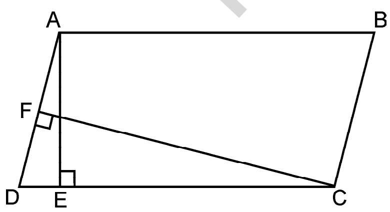
2. Area of a parallelogram is the product of its any side and the corresponding altitude.

3. Parallelograms on the same base and having equal areas lie between the same parallels.

4. If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle, is half the area of the parallelogram.

TEXTBOOK'S EXERCISE 9.2

Q.1. *In the figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.* **[2010]**



Sol. Area of parallelogram ABCD

$= AB \times AE$

$= 16 \times 8 \text{ cm}^2 = 128 \text{ cm}^2$

Also, area of parallelogram ABCD

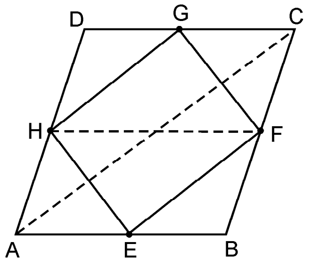
$= AD \times FC = (AD \times 10) \text{ cm}^2$

$\therefore\ AD \times 10 = 128$

$\Rightarrow \quad AD = \frac{128}{10} = 12.8 \text{ cm}$

Q.2. *If E, F, G, and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).*

Sol. Given : A parallelogram ABCD. E, F, G, H are mid-points of sides AB, BC, CD, DA respectively



To Prove : ar (EFGH) = $\frac{1}{2}$ar (ABCD)

Construction : Join AC and HF.

Proof : In ΔABC,

E is the mid-point of AB.

F is the mid-point of BC.

$\Rightarrow$ EF $\parallel$ AC and EF = $\frac{1}{2}$ AC ... (i)

Similarly, in ΔADC, we can show that

HG $\parallel$ AC and HG = $\frac{1}{2}$ AC ... (ii)

From (i) and (ii)

EF $\parallel$ HG and EF = HG

$\therefore$ EFGH is a parallelogram.

[One pair of opposite sides is equal and parallel]

In quadrilateral ABFH, we have

HA = FB and HA $\parallel$ FB

[AD = BC $\Rightarrow$ $\frac{1}{2}$AD = $\frac{1}{2}$BC $\Rightarrow$ HA = FB]

$\therefore$ ABFH is a parallelogram. [One pair of opposite sides is equal and parallel]

Now, triangle HEF and parallelogram HABF are on the same base HF and between the same parallels HF and AB.

$\therefore$ Area of ΔHEF = $\frac{1}{2}$ area of HABF ... (iii)

Similarly, area of ΔHGF = $\frac{1}{2}$ area of HFCD ... (iv)

Adding (iii) and (iv),

Area of ΔHEF + area of ΔHGF

= $\frac{1}{2}$ (area of HABF + area of HFCD)

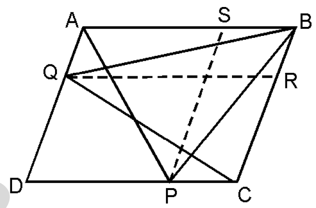
$\Rightarrow$ ar (EFGH) = $\frac{1}{2}$ar (ABCD) **Proved.**

Q.3. *P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).* **[2011 (T-II)]**

Sol. Given : A parallelogram ABCD. P and Q are any points on DC and AD respectively.

To prove : ar (APB) = ar (BQC)

Construction : Draw PS $\parallel$ AD and QR $\parallel$ AB.



Proof : In parallelogram ABRQ, BQ is the diagonal.

$\therefore$ area of ΔBQR = $\frac{1}{2}$ area of ABRQ ... (i)

In parallelogram CDQR, CQ is a diagonal.

$\therefore$ area of ΔRQC = $\frac{1}{2}$ area of CDQR ... (ii)

Adding (i) and (ii), we have

area of ΔBQR + area of ΔRQC

= $\frac{1}{2}$ [area of ABRQ + area of CDQR]

$\Rightarrow$ area of ΔBQC = $\frac{1}{2}$ area of ABCD ... (iii)

Again, in parallelogram DPSA, AP is a diagonal.

$\therefore$ area of ΔASP = $\frac{1}{2}$ area of DPSA ... (iv)

In parallelogram BCPS, PB is a diagonal.

$\therefore$ area of ΔBPS = $\frac{1}{2}$ area of BCPS ... (v)

Adding (iv) and (v)

area of ΔASP + area of ΔBPS

$$= \frac{1}{2}(\text{area of DPSA + area of BCPS})$$

$\Rightarrow$ area of ΔAPB = $\frac{1}{2}$ (area of ABCD) ... (vi)

From (iii) and (vi), we have

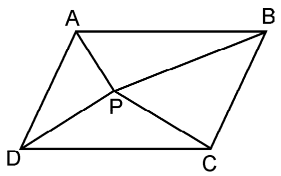
area of ΔAPB = area of ΔBQC. **Proved.**

Q.4. *In the figure, P is a point in the interior of a parallelogram ABCD. Show that*

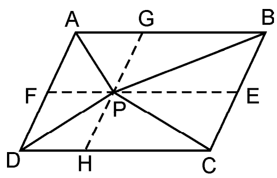
(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ *ar (ABCD)*

(ii) ar (APD) + ar (PBC)

= ar(APB) + ar (PCD) **[2011 (T-II)]**



Sol. Given : A parallelogram ABCD. P is a point inside it.



To prove :

(i) ar (APB) + ar(PCD) = $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC)

= ar (APB) + ar (PCD)

Construction : Draw EF through P parallel to AB, and GH through P parallel to AD.

Proof : In parallelogram FPGA, AP is a diagonal,

$\therefore$ area of ΔAPG = area of ΔAPF ... (1)

In parallelogram BGPE, PB is a diagonal,

$\therefore$ area of ΔBPG = area of ΔEPB ... (2)

In parallelogram DHPF, DP is a diagonal,

$\therefore$ area of ΔDPH = area of ΔDPF ... (3)

In parallelogram HCEP, CP is a diagonal,

$\therefore$ area of ΔCPH = area of ΔCPE ... (4)

Adding (1), (2), (3) and (4)

area of ΔAPG + area of ΔBPG

+ area of ΔDPH + area of ΔCPH

= area of ΔAPF + area of ΔEPB

+ area of ΔDPF + area ΔCPE

$\Rightarrow$ [area of ΔAPG + area of ΔBPG]

+ [area of ΔDPH + area of ΔCPH]

= [area of ΔAPF + area of ΔDPF]

+ [area of ΔEPB + area of ΔCPE]

$\Rightarrow$ area of ΔAPB + area of ΔCPD

= area of ΔAPD + area of ΔBPC ... (5)

But area of parallelogram ABCD

= area of ΔAPB + area of ΔCPD

+ area of ΔAPD + area of ΔBPC ... (6)

From (5) and (6)

area of ΔAPB + area of ΔPCD

= $\frac{1}{2}$ area of ABCD

or, ar (APB) + ar (PCD)

= $\frac{1}{2}$ ar (ABCD) **Proved.**

Also, from (5),

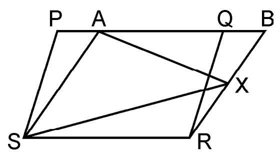
$\Rightarrow$ ar (APD) + ar (PBC)

= ar (APB) + ar (CPD) **Proved.**

Q.5. *In the figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that*

(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ *ar (PQRS)*



Sol. Given : PQRS and ABRS are parallelograms and X is any point on side BR.

To prove : (i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS)

Proof : (i) In ΔASP and BRQ, we have

∠SPA = ∠RQB [Corresponding angles] ...(1)

∠PAS = ∠QBR [Corresponding angles] ...(2)

∴ ∠PSA = ∠QRB

[Angle sum property of a triangle] ...(3)

Also, PS = QR [Opposite sides of the parallelogram PQRS] ...(4)

So, ΔASP ≅ ΔBRQ

[ASA axiom, using (1), (3) and (4)]

Therefore, area of ΔPSA = area of ΔQRB

[Congruent figures have equal areas] ...(5)

Now, ar (PQRS) = ar (PSA) + ar (ASRQ)

= ar (QRB) + ar (ASRQ)

= ar (ABRS)

So, ar (PQRS) = ar (ABRS) **Proved.**

(ii) Now, ΔAXS and ||gm ABRS are on the same base AS and between the same parallels AS and BR

∴ area of ΔAXS = $\frac{1}{2}$ area of ABRS

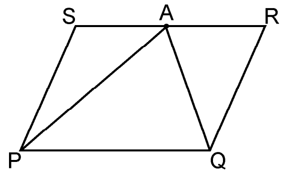
⇒ area of ΔAXS = $\frac{1}{2}$ area of PQRS

[∵ ar (PQRS) = ar (ABRS)]

⇒ ar (AXS) = $\frac{1}{2}$ ar (PQRS) **Proved.**

Q.6. *A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?* **[HOTS]**

Sol. The field is divided in three triangles.



Since triangle APQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.

∴ ar (APQ) = $\frac{1}{2}$ ar (PQRS)

⇒ 2ar (APQ) = ar(PQRS)

But ar (PQRS) = ar(APQ) + ar (PSA) + ar (ARQ)

⇒ 2 ar (APQ) = ar(APQ) + ar(PSA) + ar (ARQ)

⇒ ar (APQ) = ar(PSA) + ar(ARQ)

Hence, area of ΔAPQ

= area of ΔPSA + area of ΔARQ.

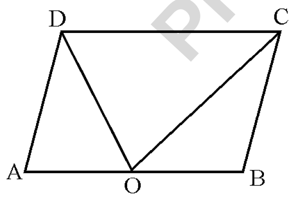
To sow wheat and pulses in equal portions of the field separately, farmer sow wheat in ΔAPQ and pulses in other two triangles or pulses in ΔAPQ and wheat in other two triangles.

OTHER IMPORTANT QUESTIONS

Q.1. *ABCD is a parallelogram and 'O' is the mid-point of AB. If area of the parallelogram is* 74 *sq cm, then area of ΔDOC is :* **[2011 (T-II)]**

(*a*) 158 *sq cm* (*b*) 37 *sq cm*

(*c*) 18.5 *sq cm* (*d*) 222 *sq cm*



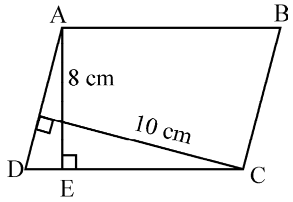
Sol. (*b*) Since, ΔDOC and parallelogram ABCD are on the same base AB and between the same parallels AB and DC, therefore,

Area of ΔDOC = $\frac{1}{2}$ area of parallelogram ABCD = $\frac{1}{2}$ × 74 sq cm = 37 sq cm

Q.2. *In the figure, ABCD is a parallelogram. AE ⊥ DC, CF ⊥ AD. If AB =* 16 *cm, AE =* 8 *cm and CF =* 10 *cm, then AD equals :* **[2011 (T-II)]**

(*a*) 12 *cm* (*b*) 15 *cm*

(*c*) 12.8 *cm* (*d*) 15.5 *cm*



Sol. (*c*) Area of parallelogram ABCD

= AB × AE = 16 × 8 = 128 cm^2

Also, area of parallelogram ABCD

= AD × FC = (AD × 10) cm^2

$\therefore$ AD × 10 = 128 ⇒ AD = 12.8 cm

Q.3. *A rectangle and a rhombus are on the same base and between the same parallels. Then the ratio of their areas is :* **[2011 (T-II)]**

(*a*) 1 : 1 (*b*) 1 : 2

(*c*) 1 : 2 (*d*) 1 : 4

Sol. (*a*) Since parallelograms on the same base and between the same parallels are equal in area. Therefore, option (*a*) is correct.

Q.4. *ABCD is a parallelogram.'O' is an interior point. If ar (AOB) + ar(DOC) = 43 sq units. then ar (|| gm ABCD) is :* **[2011 (T-II)]**

(*a*) 172 *sq units* (*b*) 176 *sq units*

(*c*) 43 *sq units* (*d*) 86 *sq units*

Sol. (*d*) ar(ΔAOB) + ar(ΔDOC)

$$= \frac{1}{2}\text{ar}(\|\text{gm ABCD})$$

⇒ ar(||gm ABCD) = 2 × 43 sq units

= 86 sq units

Q.5. *If E, F, G, H are respectively the mid-points of the sides of a parallelogram ABCD, and ar (EFGH) = 40 cm^2, then the ar (parallelogram ABCD) is :* **[2011 (T-II)]**

(*a*) 40 cm^2 (*b*) 20 cm^2

(*c*) 80 cm^2 (*d*) 60 cm^2

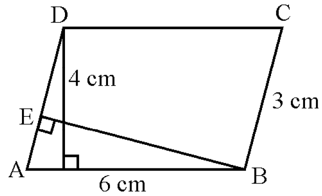
Sol. (*c*) ar (EFGH) = $\frac{1}{2}$ar(ABCD)

⇒ ar(ABCD) = 2 × 40 cm^2 = 80 cm^2

Q.6. *In the given figure, if ABCD is a parallelogram then length of BE is :* **[2011 (T-II)]**

(*a*) 24 *cm* (*b*) 26 *cm*

(*c*) 6 *cm* (*d*) 8 *cm*



Sol. (*d*) Area of parallelogram

ABCD = (6 × 4) cm^2 = 24 cm^2

Also, area of parallelogram = BE × 3 cm^2

BE × 3 = 24 ⇒ BE = 8 cm

Q.7. *If area of parallelogram ABCD is 25 cm^2 and on the same base CD, a triangle BCD is given such that area BCD = x cm^2, then value of x is :* **[2011 (T-II)]**

(*a*) 25 cm^2 (*b*) 12.5 cm^2

(*c*) 15 cm^2 (*d*) 20 cm^2

Sol. (*b*) $2x$ = 25 cm^2

$$\Rightarrow x = \frac{25}{2}\text{cm}^2 \Rightarrow x = 12.5\text{ cm}^2$$

Q.8. *The altitude of a parallelogram is twice the length of the base and its area is 1250 cm^2. The lengths of the base and the altitude respectively are :*

(*a*) 20 *cm*, 40 *cm* (*b*) 35 *cm*, 70 *cm*

(*c*) 25 *cm*, 50 *cm* (*d*) 15 *cm*, 30 *cm*

Sol. (*c*) Let the length of the base = x cm, then the altitude = $2x$ cm

Now, area of parallelogram = base × altitude

Then, 1250 cm^2 = $x \times 2x$ cm^2 ⇒ 1250 = $2x^2$ ⇒ x^2 = 625 ⇒ x = 25

Therefore, length of the base = 25 cm and the altitude = 50 cm

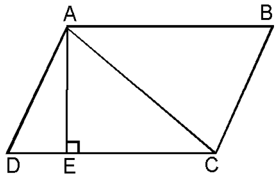
Q.9. *ABCD is a parallelogram one of whose diagonals is AC. Then, which of the following is true ?*

(a) ar (ΔADC) > ar (ΔCBA)

(b) ar (ΔADC) = ar (ΔCBA)

(c) ar (ΔABC) < ar (ΔADC)

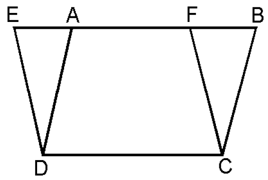
(d) none of these



Sol. (*b*) We know that diagonal of a parallelogram divides it into two triangles of equal area.

Therefore, ar (ΔACD) = ar (ΔCBA)

Q.10. *In the figure, ABCD is a parallelogram and EFCD is a rectangle. Now which of the following is correct option ?* **[HOTS]**



(a) ar (|| gm ADCF) = ar (rect. EFCD)

(b) ar (|| gm ABCD) = ar (rect. EFCD)

(c) ar (|| gm ADCF) = ar (rect. ABCD)

(d) none of these

Sol. (*b*) Parallelogram ABCD and rectangle EFCD are on the same base CD and between same parallels CD and BE.

Therefore, ar (||gm ABCD)

= ar (rect. EFCD)

Q.11. *If a triangle and a parallelogram are on the same base and between the same parallels, then the ratio of the area of the triangle to the area of parallelogram is :* **[2011 (T-II)]**

(*a*) 1 : 3 (*b*) 1 : 2

(*c*) 3 : 1 (*d*) 1 : 4

Sol. (*b*) If a triangle and a parallelogram are on the same base and between same parallels, then the area of the triangle is equal to half of the area of the parallelogram.

Therefore, ratio of the area of the triangle to the area of the parallelogram = 1 : 2.

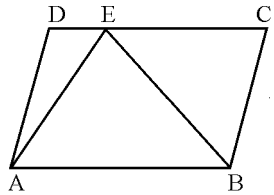
Q.12. *In the figure if area of parallelogram ABCD is* 30 cm^2*, then* **[2010]**
ar (ADE) + ar (BCE) is equal to

(*a*) 20 cm^2

(*b*) 30 cm^2

(*c*) 15 cm^2

(*d*) 25 cm^2



Sol. (*c*) Since, ΔAEB and parallelogram ABCD are on the same base AB and between the same parallels AB and DC, therefore,

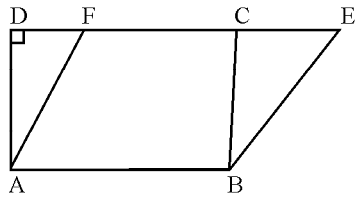
Area of ΔAEB = $\frac{1}{2}$ area of ABCD = 15 cm^2.

Now, ar(ADE) + ar(BCE)

= ar (ABCD) – ar(AEB)

= (30 – 15) cm^2 = 15 cm^2.

Q.13. *In the figure, parallelogram ABEF and rectangle ABCD have the same base AB and equal area. If AB = x, BC = y and BE = z, then :* **[2010]**



(*a*) $2(x + y) > 2(x + z)$

(*b*) $x + z < x + y$

(*c*) $x + y = x + z$

(*d*) $x + y < x + z$

Sol. (*d*) Here, parallelogram ABEF and rectangle ABCD are on the same base AB and between the same parallels AB and DC.

Also, we know that the perpendicular distance between two parallel lines is shortest.

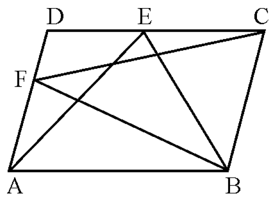
So, AD < AF or BC < BE

$\Rightarrow y < z \Rightarrow y + x < z + x$

Q.14. *In the figure, ABCD is a parallelogram, if area of ΔAEB is* 16 cm^2*, then area of ΔBFC is :*

(*a*) 32 cm^2 (*b*) 24 cm^2

(*c*) 8 cm^2 (*d*) 16 cm^2 **[2010]**



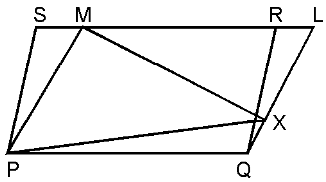
Sol. (d) Parallelogram ABCD and ΔAEB are on the same base AB and between the same parallels AB and DC.

$\therefore$ ar (ABCD) = 2 × ar(AEB) = 32 cm^2

Similarly, parallelogram ABCD and ΔBCF are on the same base BC and between the same parallels BC and AD.

$\therefore$ ar (ΔBFC) = $\frac{1}{2}$ ar(ABCD) = 16 cm^2.

Q.15. *In the figure, PQRS and PQLM are parallelograms and X is any point on side QL. The area of ΔPMX is equal to :* **[V. Imp.]**



(a) *area of ΔRQL*
(b) *area of || gm PQRS*
(c) *area of ΔSPM*
(d) $\frac{1}{2}$ *area of || gm PQLM*

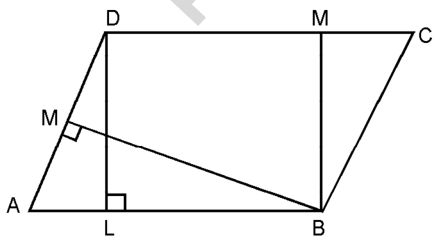
Sol. (d) ΔPMX and ||gm PQLM are on the same base PM and between the same parallels PM and QL

Therefore, area of ΔPMX

= $\frac{1}{2}$ area of ||gm PQLM

Q.16. *In the figure, the area of parallelogram ABCD is* :

(a) *AB × BM* (b) *BC × BN*
(c) *DC × DL* (d) *AD × DL*



Sol. (c) We know that the area of parallelogram = base × altitude

Therefore, area of parallelogram ABCD

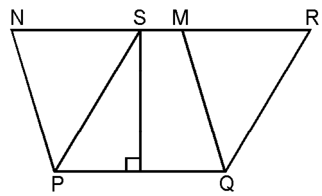
= AB × DL = DC × DL ($\because$ AB = DC)

Q.17. *If the ratio of the altitude and the area of the parallelogram is* 2 : 11, *then find the length of the base of the parallelogram.* **[2010]**

Sol. Let numerically, the altitude and the area of the parallelogram be 2x and 11x respectively. Then, 11x = Base × 2x

$\Rightarrow$ Base = $\frac{11x}{2x}$ = 5.5 units.

Q.18. *Two parallelograms PQRS and PQMN have common base PQ as shown. If PQ* = 9 *cm, SM* = 3 *cm, and ST* = 5 *cm, find the area of PQRN.* **[2010]**



Sol. Since, parallelograms PQRS and PQMN are on the same base PQ and between the same parallels PQ and NR.

$\therefore$ ar(PQRS) = ar(PQMN) = 9 × 5 cm^2

= 45 cm^2.

Also, area of trapezium PQMS

= $\frac{1}{2}$ (PQ + MS) × ST

= $\frac{1}{2}$ × (9 + 3) × 5 cm^2 = 30 cm^2.

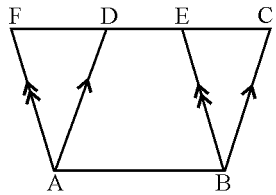
$\therefore$ ar (PQRN) = ar(PQRS) + ar (PQMN) – ar(PQMS) = (45 + 45 – 30) cm^2 = 60 cm^2.

Q.19. *Prove that the parallelograms on the same base and between the same parallels are equal in area.* **[2010, 2011 (T-II)]**

Sol. Given : Two parallelograms ABCD and ABEF are on the same base AB and between the same parallels.

To Prove : ar(||gm ABCD) = ar(||gm ABEF)

Prove : Since parallelograms are between same parallels.



∴ C, E, D and F are on same straight line.

In ΔBCE and ΔADF

∠BCE = ∠ADF [Corresponding angles]

∠BEC = ∠AFD [Corresponding angles]

∠CBE = ∠DAF [Third angles]

BC = AD

[Opposite sides of a parallelogram ABCD]

∠CBE = ∠DAF [Proved above]

BE = AF [Opposite sides of a parallelogram ABEF]

∴ ΔBCE ≅ ΔADF [SAS axiom]

Hence, ar (ΔBCE) = ar (ΔADF) ... (i)
[Congruent triangles have equal area]

Area of (||gm ABCD) [From figure]

= ar (quad ADEB) + ar (ΔADF) [From (i)]

= ar (||gm ABEF) [From figure]

Hence, ar (||gm ABCD) = ar (||gm ABEF)

Proved.

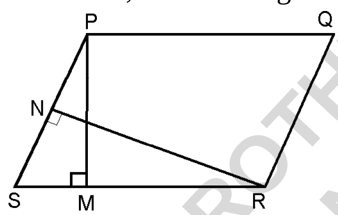
PRACTICE EXERCISE 9.2A

1 Mark Questions

Choose the correct option (Q.1 – 10) :

1. In the figure, PQRS is a parallelogram, PM ⊥ RS and RN ⊥ PS. If PQ = 12 cm,

PM = 6 cm and RN = 8 cm, then the length of PS is equal to :



(*a*) 18 cm

(*b*) 9 cm

(*c*) 4 cm

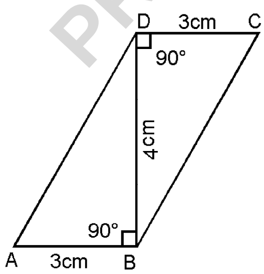
(*d*) 12 cm

2. Two adjacent sides of a parallelogram are 24 cm and 18 cm. If the distance between the longer sides is 12 cm, then the distance between the shorter sides is : **[Imp.]**

(*a*) 18 cm (*b*) 16 cm

(*c*) 9 cm (*d*) none of these

3. The area of the parallelogram ABCD in the figure is : **[Imp.]**



(*a*) 10 cm^2

(*b*) 9 cm^2

(*c*) 12 cm^2

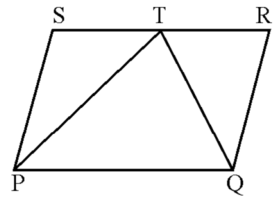
(*d*) 15 cm^2

4. If the sum of the parallel sides of a trapezium is 7 cm and distance between them is 4 cm, then area of the trapezium is :

(*a*) 28 cm^2 (*b*) 7 cm^2

(*c*) 21 cm^2 (*d*) 14 cm^2

5. In the figure, if the area of parallelogram PQRS is 172 cm^2, then the area of the traingle PTQ is : **[2010]**



(*a*) 68 cm^2

(*b*) 86 cm^2

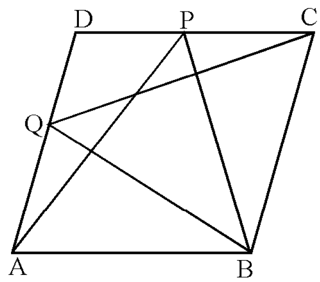
(*c*) 96 cm^2

(*d*) 72 cm^2

6. The areas of a parallelogram and a triangle are equal and they lie on the same base. If the altitude of the parallelogram is 2 cm, then the altitude of the triangle is :

(*a*) 4 cm (*b*) 1 cm (*c*) 2 cm (*d*) 3 cm

7. P and Q are any two points lying on the sides CD and AD respectively of a parallelogram ABCD. Now which of the two triangles have equal area ? **[2011 (T-II)]**



(a) ΔAPD and ΔBPC (b) ΔABQ and ΔCDQ

(c) ΔAPB and ΔBQC (d) none of these

8. The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is :

(a) a rectangle of area 24 cm^2

(b) a square of area 25 cm^2

(c) a trapezium of area 24 cm^2

(d) a rhombus of area 24 cm^2

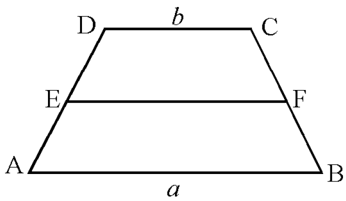
9. ABCD is a trapezium with parallel sides AB = a cm and DC = b cm. E and F are the mid-points of the non-parallel sides. The ratio of ar (ABFE) and ar (EFCD) is : **[2011 (T-II)]**

(a) $a : b$

(b) $(3a + b) : (a + 3b)$

(c) $(a + 3b) : (3a + b)$

(d) $(2a + b) : (3a + b)$



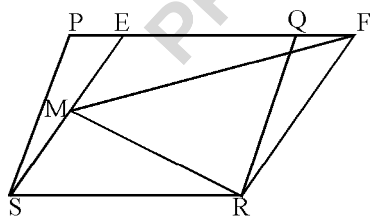
10. The area of the figure formed by joining the mid-points of the adjacent sides of a rhombus with diagonals 12 cm and 16 cm is :

(a) 48 cm^2 (b) 64 cm^2

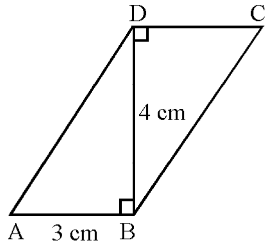
(c) 96 cm^2 (d) 192 cm^2

2 Marks Questions

11. In the figure, PQRS and EFRS are two parallelograms. Is area of ΔMFR equal to $\frac{1}{2}$ area of || gm PQRS ?

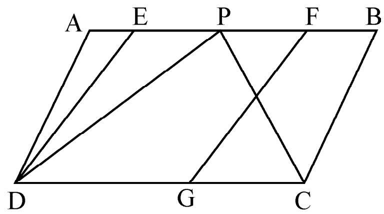


12. In the figure, ABCD is quadrilateral and BD is one of its diagonals. Show that ABCD is a parallelogram and find its area. **[Imp.]**



13. In parallelogram ABCD, AB = 10 cm. The altitude corresponding to the sides AB and AD are respectively 7 cm and 8 cm. Find AD.

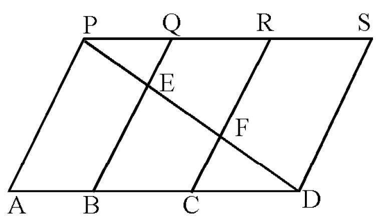
14. In the figure, ABCD and EFGD are two parallegorams and G is the mid-point of CD. Check whether area of ΔPDC is equal to half of area EFGD.



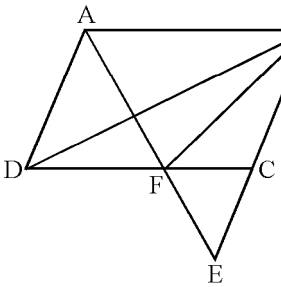
3 Marks Questions

15. If the mid-points of the sides of a quadrilateral are joined in order prove that the area of the parallelogram so formed will be half of that of the given quadrilateral.

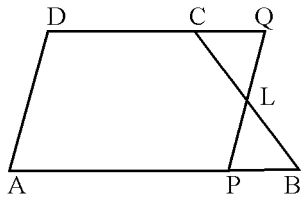
16. In the figure, PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC. Prove that ar (PQE) = ar (CFD).



17. In the figure, ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F. If ar (ΔDFB) = 3 cm^2, find the area of the parallelogram ABCD. **[2011 (T-II)]**



18. In the figure, ABCD is a trapezium in which AB || DC and L is the mid-point of BC. Through L, a line PQ || AD has been drawn which meets AB in P and DC produced to Q. Show that ar (ABCD) = ar (APQD).



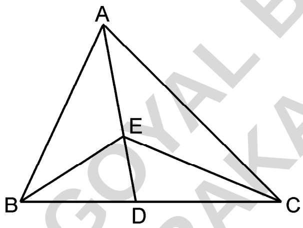
9.3 TRIANGLES ON THE SAME BASE AND BETWEEN THE SAME PARALLELS

1. Two triangles on the same base and between the same parallels are equal in area.
2. Two triangles having the same base and equal areas lie between the same parallels.
3. Area of a triangle is half the product of its base and the corresponding altitude (or height).
4. A median of a triangle divides it into two triangles of equal areas.

TEXTBOOK'S EXERCISE 9.3

Q.1. *In the figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE).* **[2011 (T-II)]**

Sol. Given : A triangle ABC, whose one median is AD. E is a point on AD.



To Prove : ar (ABE) = ar (ACE)

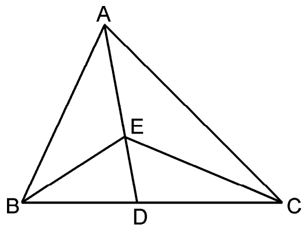
Proof : Area of ΔABD = Area of ΔACD ... (i)
(Median divides the triangle into two equal parts)

Again, in ΔEBC, ED is the median, therefore,
Area of ΔEBD = area of ΔECD .... (ii)
(Median divides the triangle into two equal parts)

Subtracting (ii) from (i), we have
area of ΔABD – area of ΔEBD
= area of ΔACD – area of ΔECD
⇒ area of ΔABE = area of ΔACE
⇒ ar (ABE) = ar (ACE) **Proved.**

Q.2. *In a triangle ABC, E is the mid-point on median AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC).* **[2011 (T-II)]**

Sol. Given : A triangle ABC, in which E is the mid-point of median AD.



To Prove : ar(BED) = $\frac{1}{4}$ ar(ABC)

Proof : In ΔABC, AD is the median.

$\therefore$ area of ΔABD = area of ΔADC ... (i)

[Median divides the triangle into two equal parts]

Again, in ΔADB, BE is a median.

$\therefore$ area of ΔABE = area of ΔBDE ... (ii)

From (i), we have

area of $\Delta ABD = \frac{1}{2}$ area of ΔABC ... (iii)

From (ii), we have

area of $\Delta BED = \frac{1}{2}$ area of ΔABD ... (iv)

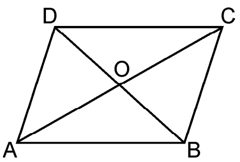
From (iii) and (iv), we have

area of $\Delta BED = \frac{1}{2} \times \frac{1}{2}$ area of ΔABC

$\Rightarrow$ area of $\Delta BED = \frac{1}{4}$ area of ΔABC

$\Rightarrow$ ar (BED) = $\frac{1}{4}$ ar(ABC) **Proved.**

Q.3. *Show that the diagonals of a parallelogram divide it into four triangles of equal area.* **[2010]**



Sol. Given : A parallelogram ABCD.

To Prove : area of ΔAOB = area of ΔBOC = area of ΔCOD = area of ΔAOD

Proof : AO = OC and BO = OD

(Diagonals of a parallelogram bisect each other)

In ΔABC, O is the mid-point of AC, therefore, BO is a median.

$\therefore$ area of ΔAOB = area of ΔBOC ... (i)

(Median of a triangle divides it into two equal parts)

Similarly, in ΔCBD, O is mid-point of DB, therefore, OC is a median.

$\therefore$ area of ΔBOC = area of ΔDOC ... (ii)

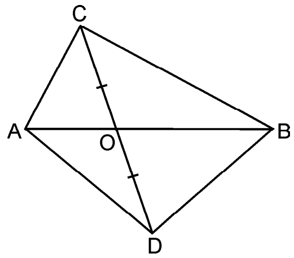
Similarly, in ΔADC, O is mid-point of AC, therefore, DO is a median.

$\therefore$ area of ΔCOD = area of ΔDOA ... (iii)

From (i), (ii) and (iii), we have

area of ΔAOB = area of ΔBOC = area of ΔDOC = area of ΔAOD **Proved.**

Q.4. *In the figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).* **[2011 (T-II)]**



Sol. Given : ABC and ABD are two triangles on the same base AB and line segment CD is bisected by AB at O.

To Prove : ar (ABC) = ar (ABD)

Proof : In ΔACD, we have

CO = OD (Given)

$\therefore$ AO is a median.

$\therefore$ area of ΔAOC = area of ΔAOD ... (i)

(Median of a triangle divides it into two equal parts)

Similarly, in ΔBCD, OB is median

$\therefore$ area of ΔBOC = area of ΔBOD ... (ii)

Adding (i) and (ii), we get

area of ΔAOC + area of ΔBOC

= area of ΔAOD + area of ΔBOD

$\Rightarrow$ area of ΔABC = area of ΔABD

$\Rightarrow$ ar (ABC) = ar (ABD) **Proved.**

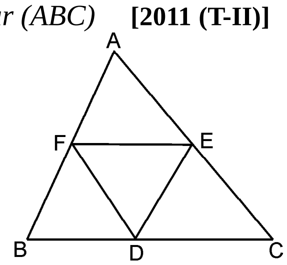
Q.5. *D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that*

(i) BDEF is a parallelogram.

(ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC) **[2011 (T-II)]**

Sol. Given : D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC.



To Prove : (i) BDEF is a parallelogram.

(ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)

Proof : (i) In ΔABC, E is the mid-point of AC and F is the mid-point of AB.

$\therefore$ EF || BC or EF || BD

Similarly, DE || BF.

$\therefore$ BDEF is a parallelogram ... (1)

(ii) Since, DF is a diagonal of parallelogram BDEF.

Therefore, area of ΔBDF

= area of ΔDEF ...(2)

Similarly, area of ΔAFE = area of ΔDEF ... (3)

and area of ΔCDE = area of ΔDEF ... (4)

From (2), (3) and (4), we have

area of ΔBDF = area of ΔAFE = area of ΔCDE

= area of ΔDEF ... (5)

Again ΔABC is divided into four non-overlapping triangles BDF, AFE, CDE and DEF.

$\therefore$ area of ΔABC = area of ΔBDF + area of ΔAFE + area of ΔCDE + area of ΔDEF

= 4 area of ΔDEF ... (6) [Using (5)]

$\Rightarrow$ area of ΔDEF = $\frac{1}{4}$ area of ΔABC

$\Rightarrow$ ar (DEF) = $\frac{1}{4}$ ar (ABC) **Proved.**

(iii) Now, area of parallelogram BDEF

= area of ΔBDF + area of ΔDEF

= 2 area of ΔDEF

= $2 \cdot \frac{1}{4}$ area of ΔABC

= $\frac{1}{2}$ area of ΔABC

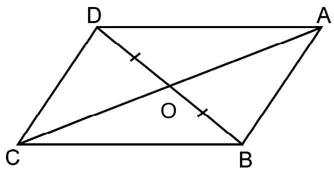
Hence, ar (BDEF) = $\frac{1}{2}$ ar (ABC) **Proved.**

Q.6. *In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that :*

(i) ar (DOC) = ar(AOB) **[2011 (T-II)]**

(ii) ar (DCB) = ar (ACB)

(iii) DA || CD or ABCD is a parallelogram.



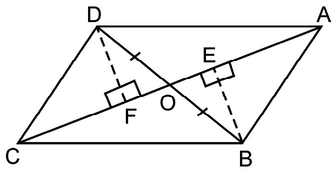
Sol. Given : Diagonal AC and BD of quadrilateral ABCD intersect at O such that OB = OD and AB = CD.

To Prove : (i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

Construction : Draw perpendiculars DF and BE on AC.



Proof : (i) area of ΔDCO = $\frac{1}{2}$ CO × DF ... (1)

area of ΔABO = $\frac{1}{2}$ AO × BE ... (2)

In ΔBEO and ΔDFO, we have

BO = DO [Given]

$\angle$BOE = $\angle$DOF [Vertically opposite angles]

$\angle$BEO = $\angle$DFO [Each = 90°]

$\Rightarrow$ ΔBOE $\cong$ ΔDOF [AAS congruence]

$\Rightarrow$ BE = DF [CPCT] ... (3)

OE = OF [CPCT] ... (4)

In ΔABE and ΔCDF, we have,

AB = CD [Given]

BE = DF [Proved above]

$\angle$AEB = $\angle$CFD [Each = 90°]

$\therefore$ ΔABE $\cong$ ΔCDF [RHS congruence]

$\Rightarrow$ AE = CF [CPCT] ... (5)

From (4) and (5), we have

OE + AE = OF + CF

$\Rightarrow$ AO = CO ... (6)

Hence, ar (DOC) = ar (AOB).

[From (1), (2), (3) and (6)] **Proved.**

(ii) In quadrilateral ABCD, AC and BD are its diagonals, which intersect at O.

Also, BO = OD [Given]

AO = OC [Proved above]

⇒ ABCD is a parallelogram

[Diagonals of a quadrilateral bisect each other]

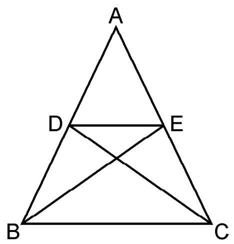
⇒ BC ∥ AD.

So, ar (DCB) = ar (ACB) [ΔDCB and ΔACB are on the same base BC and between the same parallels BC and AD] **Proved.**

(iii) In (ii), we have proved that ABCD is a parallelogram.

Q.7. *D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE ∥ BC.* **[2010]**

Sol. Given : D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC)



To Prove : DE ∥ BC

Proof : ar (DBC) = ar (EBC) [Given]

Also, triangles DBC and EBC are on the same base BC.

∴ they are between the same parallels

i.e., DE ∥ BC **Proved.**

[Two triangles having the same base and equal areas lie between the same parallels]

Q.8. *XY is a line parallel to side BC of a triangle ABC. If BE ∥ AC and CF ∥ AB meet XY at E and F respectively, show that*

ar (ABE) = ar (ACF) **[2011 (T-II)]**

Sol. Given : XY is a line parallel to side BE of a ΔABC.

BE ∥ AC and CF ∥ AB

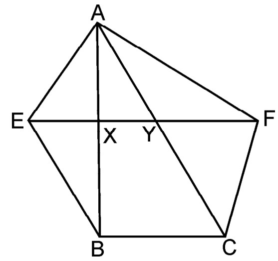
To Prove : ar (ABE) = ar (ACF)

Proof : ΔABE and parallelogram BCYE are on the same base BE and between the same parallels BE and AC.

∴ ar (ABE) = $\frac{1}{2}$ ar (BCYE) ... (i)

Similarly,

ar (ACF) = $\frac{1}{2}$ ar (BCFX) ... (ii)



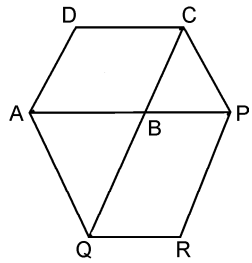
But parallelogram BCYE and BCFX are on the same base BE and between the same parallels BC and EF.

∴ ar (BCYE) = ar (BCFX) ... (iii)

From (i), (ii) and (iii), we get

ar (ABE) = ar (ACF) **Proved.**

Q.9. *The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see figure,). Show that ar (ABCD) = ar (PBQR).* **[2011 (T-II)]**



Sol. Given : ABCD is a parallelogram.

CP ∥ AQ, BP ∥ QR, BQ ∥ PR

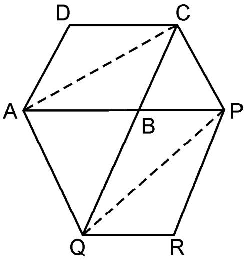
To Prove : ar (ABCD) = ar (PBQR)

Construction : Join AC and PQ.

Proof : AC is a diagonal of parallelogram ABCD.

∴ area of ΔABC = $\frac{1}{2}$ area of ABCD ... (i)

(A diagonal divides the parallelogram into two parts of equal area)



Similarly, area of ΔPBQ

$= \frac{1}{2}$ area of PBQR ... (ii)

Now, triangles AQC and AQP are on the same base AQ and between the same parallels AQ and CP.

∴ area of ΔAQC = area of ΔAQP ... (iii)

Subtracting area of ΔAQB from both sides of (iii),

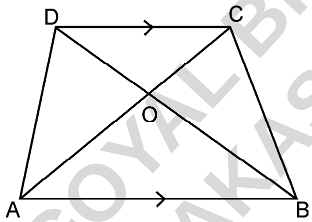
area of ΔAQC – area of ΔAQB
= area of ΔAQP – area of ΔAQB

⇒ area of ΔABC = area of ΔPBQ ... (iv)

⇒ $\frac{1}{2}$ area of ABCD = $\frac{1}{2}$ area of PBQR
[From (i) and (ii)]

⇒ area of ABCD = area of PBQR **Proved.**

Q.10. *Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC)*
[2010, 2011 (T-II)]



Sol. Given : Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

To Prove : ar (AOD) = ar (BOC)

Proof : Triangles ABC and BAD are on the same base AB and between the same parallels AB and DC.

∴ area of ΔABC = area of ΔBAD

⇒ area of ΔABC – area of ΔAOB
= area of ΔABD – area of ΔAOB
(Subtracting area of ΔAOB from both sides)

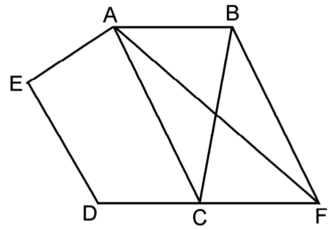
⇒ area of ΔBOC = area of ΔAOD
(From figure)

Hence, ar (BOC) = ar (AOD) **Proved.**

Q.11. *In the figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that* **[2011 (T-II)]**

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)



Sol. Given : ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F.

To Prove : (i) ar (ACB) = ar (ACF)
(ii) ar (AEDF) = ar (ABCDE)

Proof : (i) ΔACB and ΔACF lie on the same base AC and between the same parallels AC and BF.

Therefore, ar (ACB) = ar (ACF) **Proved.**

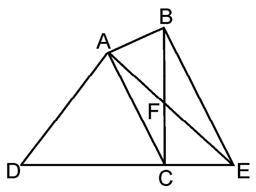
(ii) We have, from (i), ar (ACB = ar (ACE)

So, ar (ACB) + ar (ACDE) = ar (ACF) + ar (ACDE)(Adding same areas on both sides)

⇒ ar (ABCDE) = ar(AEDF) **Proved.**

Q.12. *A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.*

Sol. ABCD is the plot of land in the shape of a quadrilateral. From B draw BE ||AC to meet DC produced at E.



To Prove : ar (ABCD) = ar (ADE)

Proof : ΔBAC and ΔEAC lie on the same base AC and between the same parallels AC and BE.

Therefore, ar (BAC) = ar (EAC)

So, ar (BAC) + ar (ADC)

= ar (EAC) + ar (ADC)

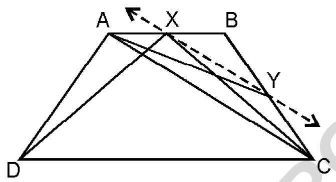
[Adding same area on both sides]

Or, ar (ABCD) = ar (ADE)

Hence, the Gram Panchayat took over ΔAFB and gave ΔEFC.

Q.13. *ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).* **[2011 (T-II)]**

Sol. Given : ABCD is a trapezium with AB || DC. AC || XY.



To Prove : ar(ΔADX) = ar(ΔACY).

Construction : Join XC

Proof : Since AB || DC ∴ AX || DC

⇒ ar (ADX) = ar (AXC) ... (i)

(Having same base AX and between the same parallels AX and DC)

Since, AC || XY

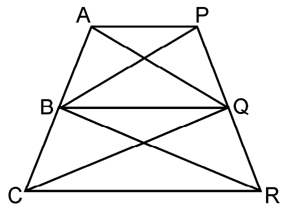
⇒ ar (AXC) = ar (ACY) ... (ii)

(Having same base AC and between the same parallels AC and XY)

⇒ ar (ADX) = ar (ACY)

(From (i), (ii)) **Proved.**

Q.14. *In the figure, AP || BQ || CR. Prove that ar(AQC) = ar (PBR).* **[2011 (T-II)]**



Sol. Given : In figure, AP || BQ || CR.

To Prove : ar (AQC) = ar (PBR)

Proof : Triangles ABQ and PBQ are on the same base BQ and between the same parallels AP and BQ.

∴ ar (ABQ) = ar (PBQ) ... (1)

(Triangles on the same base and between the same parallels are equal in area)

Similarly, triangles BQC and BQR on the same base BQ and between the same parallels BQ and CR

∴ ar (BQC) = ar (BQR) ... (2)

[Same reason]

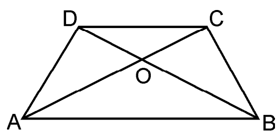
Adding (1) and (2), we get

ar (ABQ) + ar (BQC) = ar (PBQ) + ar (BQR)

⇒ ar (AQC) = ar (PBR). **Proved.**

Q.15. *Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.* **[2011 (T-II)]**

Sol. Given : Diagonals AC and BD of a quadrilateral ABCD intersect at O, such that ar (AOD) = ar (BOC)



To Prove : ABCD is a trapezium.

Proof : ar (ΔAOD) = ar(ΔBOC)

⇒ ar(AOD) + ar(BOA)

= ar (BOC) + ar (BOA)

⇒ ar(ABD) = ar(ABC)

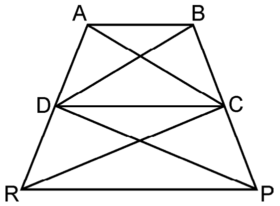
But, triangle ABD and ABC are on the same base AB and have equal area.

∴ they are between the same parallels, i.e., AB || DC. Hence, ABCD is a trapezium.

[∵ A pair of opposite sides is parallel]

Proved.

Q.16. *In the figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.* **[2011 (T-II)]**



Sol. Given : ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC)

To Prove : ABCD and DCPR are trapeziums.

Proof : ar (BDP) = ar (ARC)

$\Rightarrow$ ar (DPC) + ar (BCD)

= ar (DRC) + ar (ACD)

$\Rightarrow$ ar (BCD) = ar (ACD)

[$\because$ ar (DRC) = ar (DPC)]

But, triangles BCD and ACD are on the same base CD.

$\therefore$ They are between the same parallels, i.e., AB || DC

Hence, ABCD is a trapezium. ... (i)

Proved.

Also, ar (DRC) = ar (DPC) [Given]

Since, triangles DRC and DPC are on the same base CD.

$\therefore$ they are between the same parallels,

i.e., DC || RP

Hence, DCPR is a trapezium ... (ii) **Proved.**

OTHER IMPORTANT QUESTIONS

Q.1. *Which of the following is true ?*

(*a*) *Area of a triangle = Base × Altitude*

(*b*) *Altitude of a triangle =* $\dfrac{Area}{Base}$

(*c*) *Base of triangle =* $2 \times \dfrac{Area}{Altitude}$

(*d*) *none of these*

Sol. (*c*) Area of triangle

$$= \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\Rightarrow \text{base of triangle} = \frac{2 \times \text{area}}{\text{altitude}}$$

Q.2. *The median of a triangle divides it into two :* **[V. Imp.]**

(*a*) *triangles of equal area*

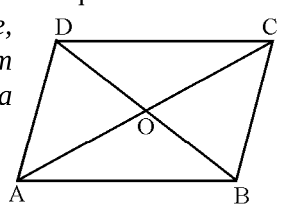
(*b*) *congruent triangles*

(*c*) *right triangles*

(*d*) *isosceles triangles*

Sol. (*a*) The median of a triangle divides the triangle into two triangles of equal area.

Q.3. *In the given figure, ABCD is a parallelogram whose area is 20 cm^2. Area of ΔAOD is :* **[2010, 2011 (T-II)]**



(*a*) 10 cm^2 (*b*) 15 cm^2

(*c*) 5 cm^2 (*d*) 12 cm^2

Sol. (*c*) We know that the diagonals of a parallelogram divide it into four triangles of equal area.

$\therefore$ Area of ΔAOD

$$= \frac{1}{4} \times \text{area of ABCD} = 5 \text{ cm}^2.$$

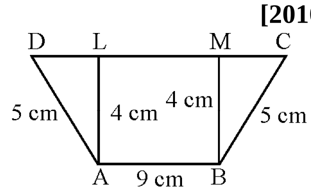
Q.4. *In the given figure, area of trapezium ABCD is :* **[2010]**

(*a*) 38 *cm*^{2}

(*b*) 20 *cm*^{2}

(*c*) 48 *cm*^{2}

(*d*) 60 *cm*^{2}



Sol. (*c*) $DL^2 = AD^2 - AL^2$

[Pythagoras theorem]

$\Rightarrow DL = \sqrt{25-16}$ cm = 3 cm.

$\therefore$ Area of ΔADC $= \dfrac{1}{2} \times AL \times DL$

$$= \frac{1}{2} \times 4 \times 3 \text{ cm}^2 = 6 \text{ cm}^2$$

Similarly, area of ΔBMC = 6 cm^2

∴ area of trapezium ABCD

= area of rectangle ABML + area of ΔADL + area of ΔBMC

= (9 × 4 + 6 + 6) cm^2 = 48 cm^2

Q.5. *AD is a median of ABC. If area of ΔABC is* 50 cm^2*, then area of ΔABD is :* **[2010]**

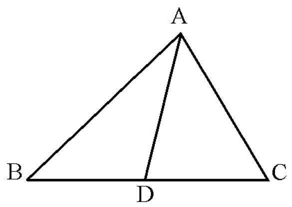
(*a*) 100 cm^2 (*b*) 25 cm^2

(*c*) 50 cm^2 (*d*) 75 cm^2

Sol. (*b*) We know that a median of a triangle divides it into two triangles of equal area.

$$\therefore \text{ Area of } \Delta ABD = \frac{1}{2} \times \text{area of } \Delta ABC$$

$$= \frac{1}{2} \times 50 \text{ cm}^2 = 25 \text{ cm}^2.$$



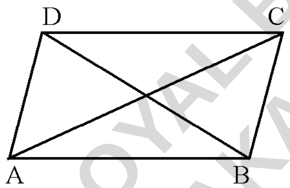
Q.6. *If ABCD is a parallelogram, then which of the following is true ?* **[2011 (T-II)]**

(*a*) *ar (ΔABD) = ar (ΔBCD)*

(*b*) *ar (ΔABD) = ar (ΔABC)*

(*c*) *ar (ΔABC) = ar (ΔACD)*

(d) *all are true*



Sol. (*d*) We know that diagonal of a parallelogram divides it into two triangles of equal area.

In the figure, diagonal BD divides ||gm ABCD in ΔABD and ΔBCD.

Therefore, ar (ΔABD) = ar (ΔBCD)

Similarly, diagonal AC divides ||gm ABCD in ΔABC and ΔACD.

Therefore, ar (ΔABC) = ar (ΔACD)

Again, ΔABD and ΔABC are on the same base AB and between same parallels AB and CD

Therefore, ar (ΔABD) = ar (ΔABC)

Hence, all the given options are true.

Q.7. *The area of a triangle is equal to the area of a rectangle whose length and breadth are* 18 *cm and* 12 *cm respectively. If the base of the triangle is* 24 *cm, then its altitude is :* **[Imp.]**

(*a*) 18 *cm* (*b*) 24 *cm*

(*c*) 36 *cm* (*d*) 48 *cm*

Sol. (*a*) Area of rectangle

= length × breadth = (18 × 12) cm^2

And, area of triangle = $\frac{1}{2}$ × base × altitude

$$= \left(\frac{1}{2} \times 24 \times h\right) \text{cm}^2$$

Also, area of triangle = Area of rectangle

Therefore, $\frac{1}{2} \times 24 \times h = 18 \times 12$

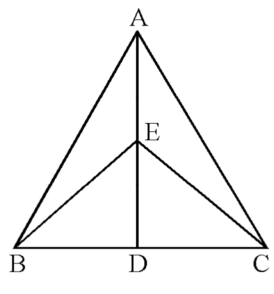
$$\Rightarrow h = \frac{2 \times 18 \times 12}{24} \Rightarrow h = 18 \text{ cm.}$$

Q.8. *In a ΔABC, E is the mid-point of median AD, then ar (ΔBED) is :*

(*a*) $\frac{1}{2}$ *ar (ΔABC)* (*b*) $\frac{1}{3}$ *ar (ΔABC)*

(*c*) $\frac{1}{4}$ *ar (ΔABC)* (*d*) *none of these*

[2010, 2011 (T-II)]



Sol. (*c*) Since, AD is the median of ΔABC, therefore,

ar (ΔABD) = $\frac{1}{2}$ ar (ΔABC) … (i)

Again, BE is the median of ΔABD.

[$\because$ E is the mid-point of AD]

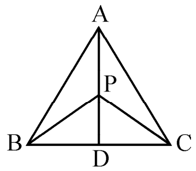
Therefore, $Ar(\Delta BED) = \frac{1}{2} ar\ (\Delta ABD)$

$= \frac{1}{2} \cdot \frac{1}{2}\ ar\ (\Delta ABC)$ [Using (i)]

$= \frac{1}{4}\ ar\ (\Delta ABC)$.

Q.9. *P is any point on the median AD of ΔABC. Show that ar(APB) = ar (ACP).* **[2010]**

Sol. Since, AD is the median of ΔABC, therefore, $ar\ (\Delta ABD) = ar\ (\Delta ACD)$ … (i)



Also, PD is the median of ΔPBC, therefore, $ar\ (\Delta PBD) = ar\ (\Delta PCD)$ … (ii)

Subtracting (ii) from (i), we get

$ar\ (\Delta ABD) - ar\ (\Delta PBD)$

$= ar\ (\Delta ACD) - ar\ (\Delta PCD)$

$\Rightarrow ar\ (\Delta APB) = ar\ (\Delta APC)$. **Proved**

Q.10. *Check whether the following statement is true. PQRS is a rectangle inscribed in a quadrant of a circle of radius* 13 *cm. A is any point on PQ. If PS* = 5 *cm, then ar (PAS)* = 30 *cm*^{2}.

Sol. False. In given figure PQ

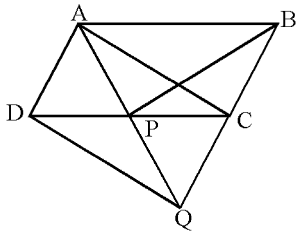
$= RS = \sqrt{169 - 25}$ cm = 12 cm.

$\therefore ar\ \Delta PQR = \frac{1}{2} \times 5 \times 12 = 30\ cm^2$

Since, the point A is between P and Q, therefore, ar (PAS) < ar (PQR)

So, the given statement is false.

Q.11. *In the figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that ar (BPC) = ar (DPQ).* **[2010]**



Sol. We have, ar (APC) = ar (BPC) … (i)

[Triangles on the same base CP and between the same parallels AB and CD]

Now, ||gm ABCD and ACQD are on the same base AD and between the same parallels AD and BQ. Therefore,

ar (||gm ABCD) = ar (||gm ACQD)

$\Rightarrow \frac{1}{2}$ ar (||gm ABCD) = $\frac{1}{2}$ ar (||gm ACQD)

$\Rightarrow$ ar (ADC) = ar (ADQ) … (ii)

$\Rightarrow$ ar (ADC) – ar (ADP)

= ar (ADQ) – ar (ADP)

$\Rightarrow$ ar (APC) = (DPQ) … (iii)

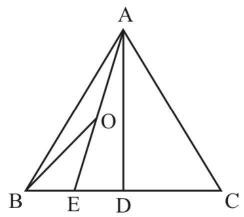
From, (i) and (iii), we get

ar (BPC) = ar (DPQ) **Proved.**

Q.12. *D is the mid-point of side BC of a ΔABC and E is the mid point of BD. If O is the mid-point of AE, then show that ar (BOE) =* $\frac{1}{8}$ *ar (ABC).*

[V. Imp.]

Sol.



Since, D is the mid-point of BC, therefore, AD is the median of ΔABC, so, $ar\ (\Delta ABD) = ar\ (ACD)$

$\Rightarrow ar\ (\Delta ADB) = \frac{1}{2}\ ar\ (\Delta ABC)$ … (i)

Similarly, AE is the median of ΔABD so, $ar\ (\Delta ABE) = ar\ (\Delta ADE)$

$\Rightarrow ar\ (\Delta ABE) = \frac{1}{2} ar\ (\Delta ABD)$

$= \frac{1}{2} \cdot \frac{1}{2}$ ar (ΔABC) [Using ... (i)]

$= \frac{1}{4}$ ar (ΔABC) … (ii)

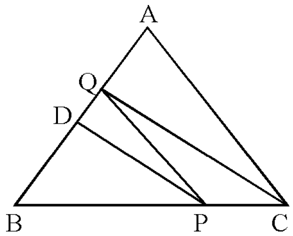
Again, BO is the median of ΔABE so,
ar (ΔBOE) = ar (ΔAOB)

$= \frac{1}{2}$ ar (ΔABE) $= \frac{1}{2} \cdot \frac{1}{4}$ ar (ΔABC)
[Using (ii)]

$= \frac{1}{8}$ ar (ΔABC)

Q.13. *In the figure, ABC is a triangle, and D is the mid-point of AB and P is any point on BC. If CQ || PD meets AB in Q, show that*

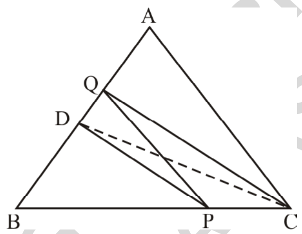
ar (BPQ) = $\frac{1}{2}$ *ar (ABC).* **[2011 (T-II)]**



Sol. Join CD.

Since, D is the mid-point of AB. So, in ΔABC, CD is the median.

ar (ΔBCD) $= \frac{1}{2}$ ar (ΔABC) … (i)



Since, Δs PDQ and PDC are on the same base PD and between the same parallels PD and QC.

ar (ΔPDQ) = ar (ΔPDC) … (ii)

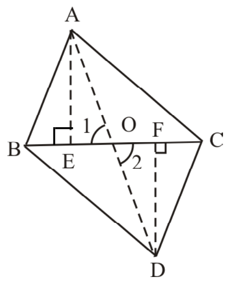
Now, from (i), ar (ΔBCD) $= \frac{1}{2}$ ar (ΔABC)

⇒ ar(ΔBPD) + ar (ΔPDC) $= \frac{1}{2}$ ar (ΔABC)

⇒ ar(ΔBPD) + ar (ΔPDQ) $= \frac{1}{2}$ ar (ΔABC)
[Using (ii)]

⇒ ar(ΔBPQ) $= \frac{1}{2}$ ar (ΔABC)

Q.14. *Triangles ABC and DBC are on the same base BC with vertices A and D on opposite sides of BC such that ar (ABC) = ar (DBC). Show that BC bisects AD.* **[2010, 2011 (T-II)]**



Sol. Since, Δs ABC and DBC are equal in area and have a common side BC. Therefore, the altitudes corresponding to BC are equal, i.e.,

AE = DF

Now, in ΔAEO and ΔDFO, we have

∠1 = ∠2 (Vertically opp. angles)

∠AEO = ∠DFO (Each = 90°)

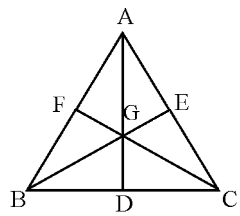
and, AE = DF

So, by AAS criterion of congruence, ΔAEO ≅ ΔDFO

⇒ AO = DO ⇒ BC bisects AD.

Q.15. *If the medians of a ΔABC intersect at G, show that ar(ΔAGB) = ar(AGC)*

= *ar(ΔBGC) =* $\frac{1}{3}$ *ar(ΔABC)* **[2011 (T-II)]**



Sol. We know that a median of a triangle divides it into two triangles of equal area.

In ΔABC, AD is the median.

∴ ar(ΔABD) = ar(ΔACD) …(i)

In ΔGBC, GD is the median.

∴ ar(ΔGBD) = ar(ΔGCD) …(ii)

From (i) and (ii), we get

ar(ΔABD) – ar(ΔGBD)

= ar(ΔACD) – ar(ΔGCD)

⇒ ar(ΔAGB) = ar(ΔAGC)

Similarly, ar(ΔAGC) = ar(ΔBGC)

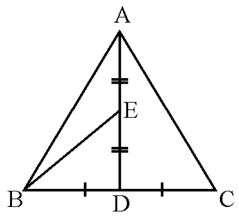
∴ ar(ΔAGB) = ar(ΔAGC) = ar(ΔBGC)…(iii)

But, ar(ΔABC) = ar(ΔAGB) + ar(ΔAGC) + ar(ΔBGC) = 3ar(ΔAGB) [Using (iii)]

∴ ar(ΔAGB) = $\frac{1}{3}$ ar(ΔABC)

Hence, ar(ΔAGB) = ar(ΔAGC) = ar(ΔBGC) = $\frac{1}{3}$ (ar ΔABC)

Q.16. *E is the mid-point of the median AD of a ΔABC. Show that ar(BED) =* $\frac{1}{4}$ *ar(ΔABC)* **[2011 (T-II)]**



Sol. Since AD is a median of ΔABC and median divides a triangle into two triangles of equal area.

∴ ar(ΔABD) = ar(ΔADC)

⇒ ar(ΔABD) = $\frac{1}{2}$ar(ΔABC) …(i)

In ΔABD, BE is the median

∴ ar(ΔBED) = ar(ΔBAE) …(ii)

⇒ ar(ΔBED) = $\frac{1}{2}$ar(ΔABD)

⇒ ar(ΔBED) = $\frac{1}{2}$ × $\frac{1}{2}$ar(ΔABC)[Using (i)]

⇒ ar(ΔBED) = $\frac{1}{4}$ar(ΔABC)

PRACTICE EXERCISE 9.3A

1 Mark Questions

Choose the correct option (Q 1 – 8) :

1. The area of a triangle is 36 cm^2 and one of its sides is 9 cm. Then, the length of the corresponding altitude to the given side is :

(*a*) 8 cm (*b*) 4 cm
(*c*) 6 cm (*d*) 9 cm

2. The area of a rhombus is 20 cm^2. If one of its diagonals is 5 cm, then the other diagonal is : **[Imp.]**

(*a*) 8 cm (*b*) 5 cm
(*c*) 4 cm (*d*) 10 cm

3. The sum of the lengths of bases of a trapezium is 13.5 cm and its area is 54 cm^2. The altitude of the trapezium is :

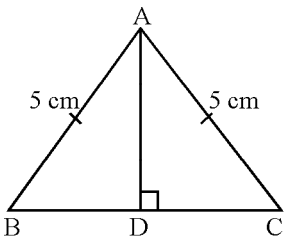
(*a*) 9 cm (*b*) 6 cm
(*c*) 8 cm (*d*) 12 cm

4. The area of an isosceles triangle, if its base and corresponding altitude are 6 cm and 4 cm respectively, is :

(*a*) 10 cm^2 (*b*) 24 cm^2
(*c*) 12 cm^2 (*d*) 20 cm^2

5. ABC is an isosceles triangle with each equal side 5 cm, perimeter 18 cm and height AD = 7 cm. Then, the area of the triangle ABC is : **[Imp.]**

(*a*) 30 cm^2 (*b*) 28 cm^2
(*c*) 14 cm^2 (*d*) 36 cm^2



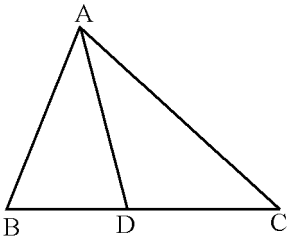
6. In the given figure, ABC is a triangle and AD is one of its medians. The ratio of areas of triangles ABD and ACD respectively is :

(*a*) 2 : 1

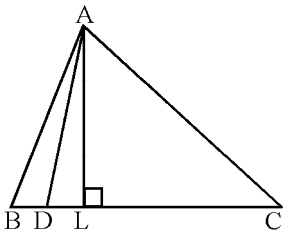
(*b*) 1 : 2

(*c*) 1 : 1

(*d*) 3 : 1



7. In the figure, the point D divides the side BC of ΔABC in the ratio $p : q$. The ratio between the ar(ΔABD) and ar(ΔADC) is :



(*a*) $\frac{p}{p+q} : \frac{q}{p+q}$ (*b*) $p : q$

(*c*) $q : p$ (*d*) none of these

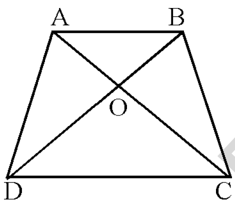
8. In the figure, ABCD is a trapezium in which AB || CD and its diagonals AC and BD intersect at O. Now ar (ΔAOD) is equal to : **[Imp.]**

(*a*) ar (ΔAOB)

(*b*) ar (ΔCOD)

(*c*) ar (ΔBOC)

(*d*) none of these

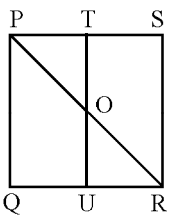


2 Marks Questions

9. ABCD is a parallelogram and X is the mid-point of AB. If ar (AXCD) = 24 cm^2, then ar(ABC) = 24 cm^2. It is true?

10. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Show that ar (BDE) = $\frac{1}{4}$ ar (ABC).

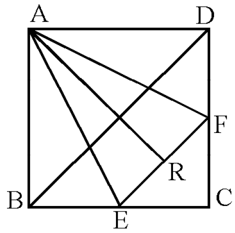
11. In the figure, PQRS is a square and T and U are respectively the mid-points of PS and QR. Find the area of ΔOTS, if PQ = 8 cm.



3 Marks Questions

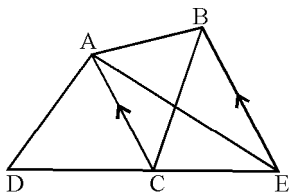
12. O is any point on the diagonal BD of a parallelogram ABCD. Show that ar (ΔOAB) = ar (ΔOBC).

13. In the figure, ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is the mid point of EF, show that ar (AER) = ar (AFR).



14. ABCD is a trapezium with parallel sides AB = *a* cm and DC = *b* cm. E and F are the mid-points of non-parallel sides. Show that ar (ABFE) : ar (EFCD) = (3*a* + *b*) : (*a* + 3*b*). **[HOTS]**

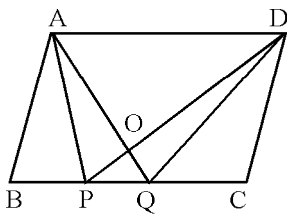
15. In the figure, ABCD is a quadrilateral and BE || AC and also BE meets DC produced at E. Show that area of ΔADE is equal to the area of the quaridlateral ABCD. **[2010]**



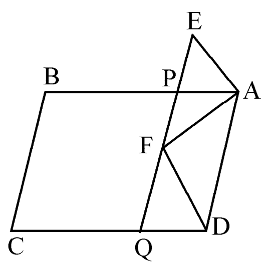
4 Marks Questions

16. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of ΔGBC = area of the quadrilateral AFGE. **[Imp.]**

17. In the figure, ABCD is a parallelogram points P and Q on BC trisect BC. Show that ar (APQ) = ar (DPQ) = $\frac{1}{6}$ ar (ABCD).



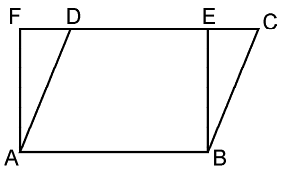
18. In the figure, ABCD and AEFD are two parallelograms. Prove that ar (PEA) = ar (QFD). **[HOTS]**



TEXT BOOK'S EXERCISE 9.4 (OPTIONAL)

Q.1. *Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.* **[2011 (T-II)]**

Sol. Given : A parallelogram ABCD and a rectangle ABEF having same base and equal area.



To Prove : 2(AB + BC) > 2(AB + BE)

Proof : Since the parallelogram and the rectangle have same base and equal area, therefore, their altitudes are equal.

Now perimeter of parallelogram ABCD.

= 2 (AB + BC) ... (i)

and perimeter of rectangle ABEF

= 2 (AB + BE) ... (ii)

In ΔBEC, $\angle BEC = 90°$

$\therefore$ $\angle BCE$ is an acute angle.

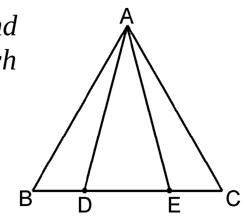
$\therefore$ BE < BC ... (iii)

(Side opposite to smaller angle is smaller)

$\therefore$ From (i), (ii) and (iii) we have

2(AB + BC) > 2(AB + BE) **Proved.**

Q.2. *In the figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).*



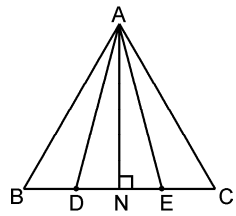
Sol.Given : A triangle ABC, in which D and E are the two points on BC such that BD = DE = EC

To Prove : ar (ABD) = ar (ADE) = ar (AEC)

Construction : Draw AN ⊥ BC

Now, ar (ABD) = $\frac{1}{2}$ × base × altitude (of ΔABD)

= $\frac{1}{2}$ × BD × AN



= $\frac{1}{2}$ × DE × AN [As BD = DE]

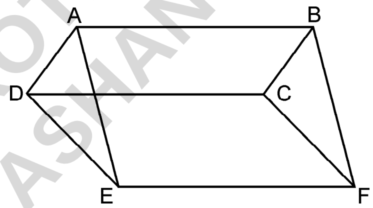
= $\frac{1}{2}$ × base × altitude (of ΔADE)

= ar (ADE)

Similarly, we can prove that ar (ADE) = ar (AEC)

Hence, ar (ABD) = ar (ADE) = ar (AEC) **Proved.**

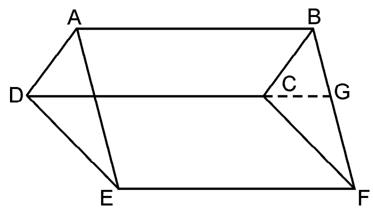
Q.3. *In the figure, ABCD, DCFE and ABFE are parallelograms. Show that ar(ADE) = ar(BCF).*



Sol. Given : Three parallelograms ABCD, DCFE and ABFE.

To Prove : ar (ADE) = ar (BCF)

Construction : Produce DC to intersect BF at G.



Proof : $\angle ADC = \angle BCG$ ... (i)

(Corresponding angles)

$\angle EDC = \angle FCG$ ... (ii)

(Corresponding angles)

$\Rightarrow$ $\angle ADC + \angle EDC = \angle BCG + \angle FCG$

[By adding (i) and (ii)]

$\Rightarrow$ $\angle ADE = \angle BCF$ ... (iii)

In ΔADE and ΔBCF, we have

AD = BC (Opposite sides of ∥ gm ABCD)

DE = CF (Opposite sides of || gm DCEF)

∠ADE = ∠BCF (From (iii))

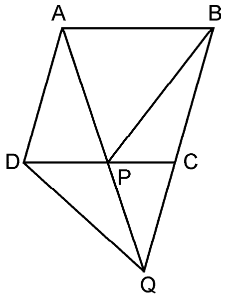
∴ ΔADE ≅ ΔBCF (SAS congruence)

⇒ ar (ADE) = ar (BCF)

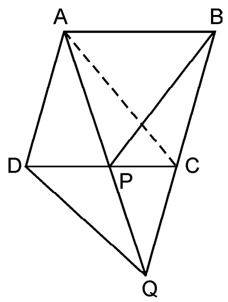
(Congruent triangles are equal in area) **Proved.**

Q.4. *In the figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that*

ar (BPC) = ar (DPQ).



Sol. Given : ABCD is a parallelogram, in which BC is produced to a point Q such that AD = CQ and AQ intersects DC at P.



To Prove : ar (BPC) = ar (DPQ)

Construction : Join AC.

Proof : Since AD || BC ⇒ AD || BQ

ar (ADC) = ar (ADQ)

(Having same base AD and between the same parallels AD and CQ)

⇒ ar (ADP) + ar (APC)

= ar (ADP) + ar (DPQ) (From figure)

⇒ ar (APC) = ar (DPQ) ... (i)

Now, since AB || DC ⇒ AB || PC

ar (APC) = ar (BPC) ... (ii)

(Having same base PC and between the same parallels AB and CD)

⇒ ar (ΔBPC) = ar (ΔDPQ) (From (i) and (ii))

Proved.

Q.5. *In the figugre, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that*

(i) ar (BDE) = $\frac{1}{4}$ *ar(ABC)*

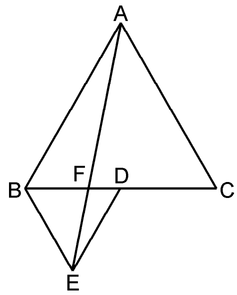
(ii) ar (BDE) = $\frac{1}{2}$ *ar(BAE)*

(iii) ar (ABC) = 2 ar(BEC)

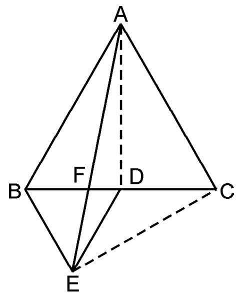
(iv) ar (BEF) = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ *ar (AFC)*



Sol. Given : ABC and BDE are equilateral triangles, D is the mid-point of BC and AE intersects BC at F.



Construction : Join AD and EC.

Proof : ∠ACB = 60° ..(1)

(Angle of an equilateral triangle)

∠EBC = 60° (Same reason)

⇒ ∠ACB = ∠EBC

⇒ AC || BE (Alternate angle are equal)

Similarly, we can prove that AB || DE ... (2)

(i) D is the mid-point of BC, so AD is a median of ΔABC

∴ ar (ABD) = $\frac{1}{2}$ ar (ABC) ... (3)

ar (DEB) = ar (DEA) (Triangles on the same base DE and between the same parallels DE and AB)

⇒ ar (DEB) = ar(ADF) + ar (DEF) ...(4)

Also, ar (DEB) = $\frac{1}{2}$ ar (BEC) (DE is a median)

= $\frac{1}{2}$ ar (BEA)

(Triangles on the same base BE and between the same parallels BE and AC) ... (5)

$\Rightarrow$ 2 ar (DEB) = ar (BEA)

$\Rightarrow$ 2 ar (DEB) = ar (ABF) + ar (BEF) ... (6)

Adding (4) and (6), we get

3 ar (DEB) = ar (ADF)

+ ar (DEF) + ar (ABF) + ar (BEF)

$\Rightarrow$ 3 ar (DEB) = ar (ADF) + ar(ABF)

+ ar (DEF) + ar (BEF)

= ar (ABD) + ar (BDE)

$\Rightarrow$ 2 ar (DEB) = ar (ABD)

$\Rightarrow$ ar (DEB) = $\frac{1}{4}$ ar (ABC) (From (3))

Proved.

(ii) From (5) above, we have

ar (BDE) = $\frac{1}{2}$ ar (BAE) **Proved.**

(iii) ar (DEB) = $\frac{1}{2}$ ar (BEC) (DE is a median)

$\Rightarrow$ $\frac{1}{4}$ ar (ABC) = ar $\frac{1}{2}$ (BEC) (From part (i))

$\Rightarrow$ ar (ABC) = 2 ar (BEC) **Proved.**

(iv) ar (DEB) = ar (DEA)

(Triangles on the same base BE and between the same parallels DE and AB) ... (7)

$\Rightarrow$ ar (DEB) – ar (DEF) = ar (DEA) – ar (DEF)

$\Rightarrow$ ar (BFE) = ar (AFD) **Proved.**

(v) ar (BDE) = $\frac{1}{4}$ ar (ABC) (From (i))

= $\frac{1}{4}$ .2. ar (ABD) = $\frac{1}{2}$ ar (ABD) …(8)

$\because$ Base of ΔBDE and ΔABD are same, so,

altitude of ΔBDE = $\frac{1}{2}$ altitude of ΔABD

$\Rightarrow$ altitude of BEF = $\frac{1}{2}$ altitude of ΔABD …(9)

Also, ar (BEF) = ar (AFD) (From (iv))…(10)

From (9) and (10), BF = 2FD

Now, in ΔBFE and ΔFED, we have

BF = 2FD and altitude of ΔBFE = altitude of ΔFED

So, ar (BFE) = 2ar (FED) **Proved.**

(vi) From (v), we have ar (FED)

= $\frac{1}{2}$ ar (BFE) = $\frac{1}{2}$ ar (AFD) (From part (iv))

Now ar (AFC) = ar (AFD) + ar (ADC)

= ar (AFD) + $\frac{1}{2}$ ar (ABC) (BE is a median)

= ar (AFD) + 2ar (BDE) (From part (i))

= ar (AFD) + 2ar (ADE)

= ar (AFD) + 2ar (AFD) + 2 ar (DEF)

= 3 ar (AFD) + ar (BFE) (From part (v))

= 3 ar (AFD) + ar (AFD) (From part (iv))

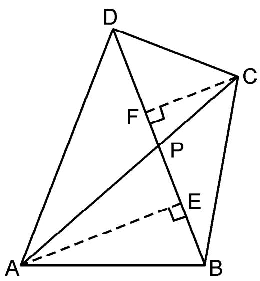
= 4 ar (AFD)

$\therefore$ $\frac{1}{8}$ ar (AFC) = $\frac{1}{2}$ ar (AFD) = ar (FED)

(From above) **Proved.**

Q.6. *Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar (APB) × ar (CPD) = ar (APD) × ar (BPC).*

Sol. Given : ABCD is a quadrilateral whose diagonals intersect each other at P.



Construction : Draw AE $\perp$ BD and CF $\perp$ BD.

Proof : ar (APB) = $\frac{1}{2}$ × PB × AE ... (i) ar (CPD) = $\frac{1}{2}$ × DP × CF ... (ii)

Now, ar (BPC) = $\frac{1}{2}$ × BP × CF ... (iii)

ar (APD) = $\frac{1}{2}$ × DP × AE ... (iv)

From (i) and (ii),

ar (APB) × ar (CPD)

= $\frac{1}{4}$ × PB × DP × AE × CF ... (v)

From (iii) and (iv), we have

ar (BPC) × ar (APD)

= $\frac{1}{4}$ × BP × DP × CF × AE ... (vi)

∴ ar (APB) × ar (CPD) = ar (BPC) × ar (APD)

[From (v) and (vi)] **Proved.**

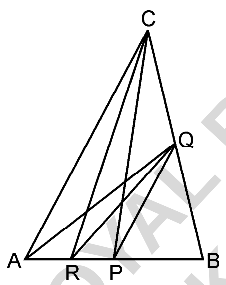
Q.7. *P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that*

(i) ar (PQR) = $\frac{1}{2}$ ar(ARC)

(ii) ar (RQC) = $\frac{3}{8}$ ar (ABC)

(iii) ar (PBQ) = ar (ARC)

Sol. Given : A triangle ABC, P and Q are mid-points of AB and BC, R is the mid point of AP.



Proof : CP is a median of ΔABC

⇒ ar (APC) = ar (PBC) = ar $\frac{1}{2}$(ABC)

Median divides a triangle into two triangles of equal area] ... (1)

CR is a median of ΔAPC

∴ ar (ARC) = ar (PRC) = $\frac{1}{2}$ ar (APC) ...(2)

QR is a median of ΔAPQ.

∴ ar (ARQ) = ar (PRQ) = $\frac{1}{2}$ ar (APQ) ...(3)

PQ is a median of ΔPBC

∴ ar (PQC) = ar (PQB) = $\frac{1}{2}$ ar (PBC) ...(4)

RQ is a median of ΔRBC

ar (RQC) = ar (RQB) = $\frac{1}{2}$ar (RBC) ...(5)

(i) ar(PQA) = ar (PQC) [Triangles on the same base PQ and between the same parallels PQ and AC]

⇒ ar (ARQ) + ar (PQR) = $\frac{1}{2}$ ar (PBC)

[From (4)]

⇒ ar (PRQ) + ar (PRQ) = $\frac{1}{2}$ ar (APC)

[From (3) and (1)]

⇒ 2 ar (PRQ) = ar (ARC) [From (2)]

⇒ ar (PRQ) = $\frac{1}{2}$ ar (ARC) **Proved.**

(ii) From (5), we have

ar (RQC) = $\frac{1}{2}$ar (RBC)

= $\frac{1}{2}$ar (PBC) + $\frac{1}{2}$ar (PRC)

= $\frac{1}{4}$ar (ABC) + $\frac{1}{4}$ar (APC) [From (1) and (2)]

= $\frac{1}{4}$ar (ABC) + $\frac{1}{8}$ar (ABC) [From (1)]

⇒ ar (RQC) = $\frac{3}{8}$ ar (ABC) **Proved.**

(iii) ar (PBQ) = $\frac{1}{2}$ar (PBC) [From (4)]

= $\frac{1}{4}$ar (ABC) [From (1)] ... (6)

ar (ARC) = $\frac{1}{2}$ar (APC) [From (2)]

= $\frac{1}{4}$ar (ABC) [From (1)] ... (7)

From (6) and (7) we have ar(PBQ)

= ar (ARC) **Proved.**

Q.8. *In the figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB* *respectively. Line segment AX ⊥ DE meets BC at Y. Show that :*

(i) ΔMBC ≅ ΔABD

(ii) ar(BYXD) = 2 ar (MBC)

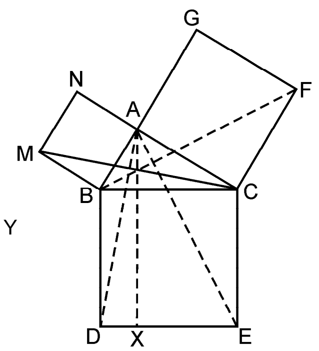
(iii) ar(BYXD) = ar (ABMN)

(iv) ΔFCB ≅ ΔACE

(v) ar(CYXE) = 2 ar (FCB)

(vi) ar(CYXE) = ar (ACFG)

(vii) ar(BCED) = ar (ABMN) + ar (ACFG)



Sol. (i) In ΔMBC and ΔABD, we have

MB = AB (Sides of a square)

BC = BD (Sides of a square)

∠MBC = ∠ABD (∠MBC = 90° + ∠ABC and ∠ABD = 90° + ∠ABC)

∴ ΔMBC ≅ ΔABD (SAS congruent)

(ii) ar (ΔMBC) ≅ ar (ABD)

(Congruent triangles have equal area)

$\Rightarrow \frac{1}{2} \times BC \times \text{height} = \frac{1}{2} \times BD \times BY$

⇒ Height of ΔMBC = BY (BC = BD)

$\therefore$ ar (MBC) = $\frac{1}{2} \times BD \times BY$

⇒ Height of ΔMBC = BY (BC = BD)

$\therefore$ ar (MBC) = $\frac{1}{2} \times BC \times BY$

⇒ 2 ar (MBC) = BC × BY ... (1)

Also, ar (BYXD) = BD × BY

= BC × BY (BC = BD) ... (2)

From (1) and (2), we have

ar(BYXD) = 2 ar (MBC) **Proved.**

(iii) ar (BYXD) = 2ar (MBC)(From part (ii))

$= 2 \times \frac{1}{2} \times$ MB × height of MBC corresponding to BC

= MB × AB (MB || NC and AB ⊥ MB)

= AB × AB (∵ AB = MB)

= AB^2

⇒ ar (BYXD) = ar (ABMN) **Proved.**

(iv) In ΔFCB and ΔACE, we have

FC = AC (Sides of a square)

∠BCF = ∠ACE (∠BCF = 90° + ∠BCA, and ∠ACE = 90° + ∠BCA)

BC = CE (Sides of a square)

ΔFCB ≅ ΔACE (SAS congruence) **Proved.**

(v) $\frac{1}{2} \times BC \times \text{height} = \frac{1}{2} \times CE \times CY$

⇒ Height of ΔFCB = CY (BC = CE)

$\therefore$ ar (FCB) = $\frac{1}{2} \times BC \times CY$

⇒ 2ar (FCB) = BC × CY ... (3)

Also, ar (CYXE) = CE × CY

= BC × CY ... (4)

From (3) and (4), we have

ar (CYXE) = 2 ar (FCB) **Proved.**

(vi) ar (CYXE) = $2 \times \frac{1}{2} \times$ FC × height of ΔFCB corresponding to FC

= FC × AC (FC || GB and AC ⊥ FC)

= AC × AC (AC = FC)

= AC^2

⇒ ar (CYXE) = ar (ACFG) **Proved.**

(vii) From (iii) and (vi), we have

ar (BYXD) + ar (CYXE)

= ar (ABMN) + ar (ACFG)

⇒ ar (BCED) = ar (ABMN) + ar (ACFG)

Proved.

B. FORMATIVE ASSESSMENT

Activity-1

Objective : To show that the area of a triangle is half the product of its base and the height using paper cutting and pasting.

Materials Required : White sheets of paper, a pair of scissors, gluestick, geometry box, etc.

Procedure :

(a) **Right angled triangle :**

1. Draw a right triangle ABC, right angled at B. Make a replica of ΔABC. Cut out both the triangles.

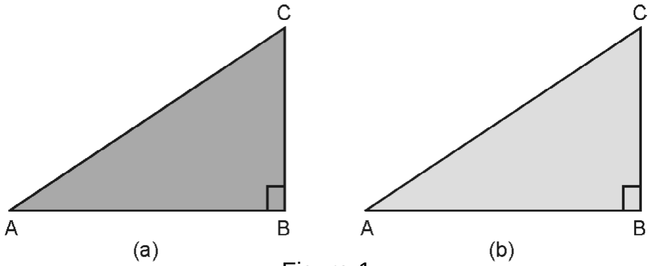


Figure-1

2. Paste the two triangular cut outs to form a rectangle as shown below.

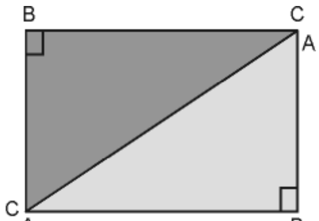


Figure-2

(b) Acute angled triangle :

1. Draw an acute angled triangle PQR on a white sheet of paper. Make a replica of ΔPQR. Cut out both the triangles.

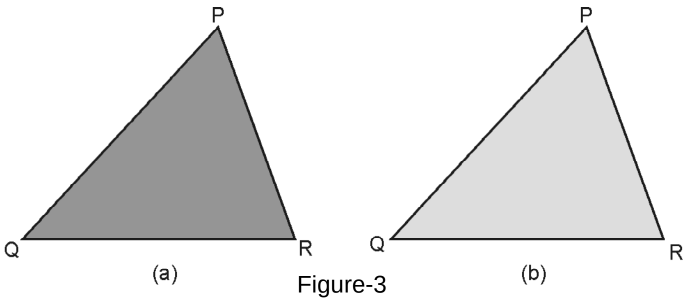


Figure-3

2. Fold the triangular cut out obtained in figure 3(b), so that the folding line passes through P and R falls on RQ. The folding line cuts QR at S. Unfold it and cut it out along the crease PS to get two triangular cut outs PSQ and PSR.

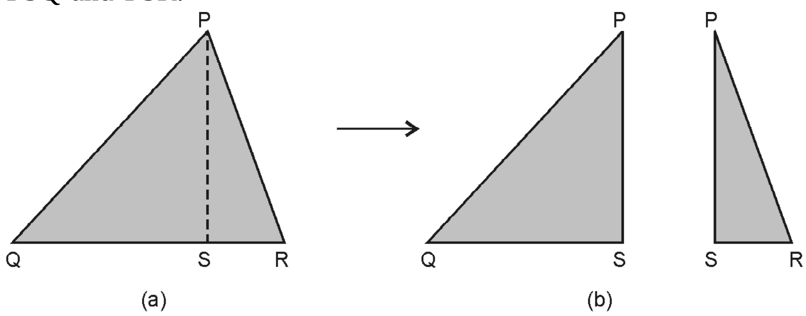


Figure-4

3. Arrange the triangular cut outs obtained in figure 3(a) and in figure 4(b), as below and paste on a white sheet of paper.

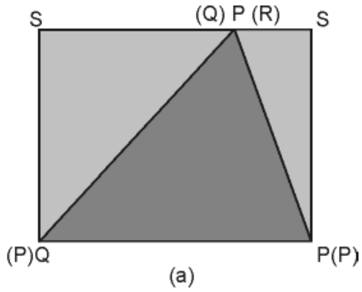


Figure-5

(c) Obtuse angled triangle

1. Draw an obtuse-angled triangle ABC on a white sheet of paper. Make a replica of ΔABC. Cut out both the triangles.

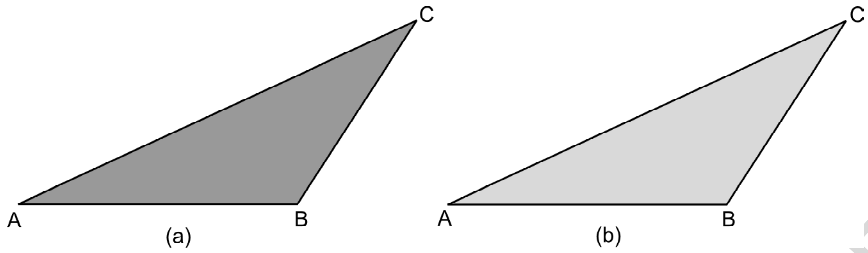


Figure-6

2. Paste the two triangular cut outs on a white sheet of paper as shown below.

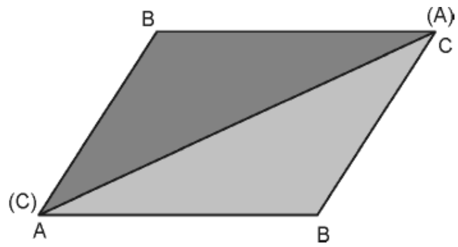


Figure-7

Observations :

1. The shape obtained in figure 2 is a rectangle having dimensions as AB and BC.
   So, area of this rectangle = AB × BC
   Also, this rectangle is obtained by combining two congruent triangles ABC
   So, area of ΔABC = Half of this rectangle
   ⇒ Area of ΔABC = $\frac{1}{2}$ × AB × BC = $\frac{1}{2}$ × base × height.
2. Figure 5 is again a rectangle, having dimensions as QR and PS.
   So, area of this rectangle = QR × PS
   Also, this rectangle is made of two congruent triangles PQR
   So, area of ΔPQR = Half of this rectangle
   ⇒ Area of ΔPQR = $\frac{1}{2}$ × QR × PS = $\frac{1}{2}$ × base × height.
   [∵ In figure 4(a), PS is the altitude of ΔPQR corresponding to the base QR]
3. Figure 7 is a parallelogram.
   Area of this parallelogram = base × height

$\Rightarrow$ Area of 2 triangles = base × height.

$\Rightarrow$ Area of a triangle = $\frac{1}{2}$ × base × height.

Conclusion : From the above activity, it is verified that.

Area of a triangle = $\frac{1}{2}$ × base × height.

Activity-2

Objective : To verify the following by activity method

A parallelogram and a rectangle standing on the same base and between the same parallels are equal in area.

Materials Required : White sheets of paper, tracing paper, colour pencils, a pair of scissors, geometry box, gluestick, etc.

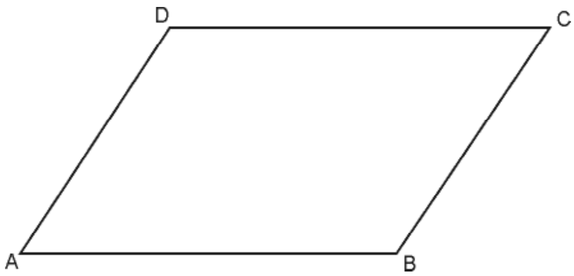


Figure-1

Procedure :

1. On a white sheet of paper, draw a parallelogram ABCD.

2. Using paper folding method, draw OB$\perp$DC.

   Colour the two parts of the parallelogram differently as shown.

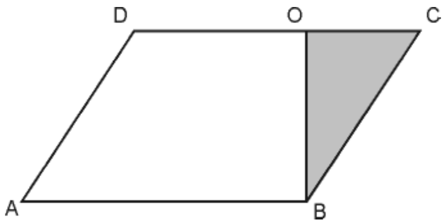


Figure-2

3. Trace the triangle OBC on a tracing paper and cut it out.

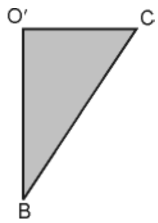


Figure-3

4. Paste the triangular cut out on the other side of the parallelogram ABCD as shown in the figure.

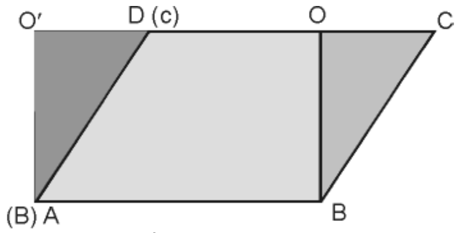


Figure-4

Observations :

1. In figure 2, area of the parallelogram ABCD = area of the trapezium ABOD + area of the ΔBCO.
2. In figure 4, ABOO´ is a rectangle.
   Area of rectangle ABOO´ = area of the trapezium ABOD + area of the triangle ADO´ (or ΔBCO)
3. From 1 and 2 above, we have, area of the parallelogram ABCD = area of the rectangle ABOO´
4. The parallelogram ABCD and the rectangle ABOO´ are on the same base AB (see figure 4)
5. Also, the parallelogram ABCD and the rectangle ABOO´ are between the same parallels AB and O´C (see figure 4)

Conclusion : From the above activity, it is verified that a parallelogarm and a rectangle standing on the same base and between the same parallels are equal in area.

Do Yourself : Draw three different parallelograms and verify the above property by activity method.

Activity-3

Objective : To verify by activity method that the parallelograms standing on the same base and between the same parallels are equal in area.

Materials Required : White sheets of paper, tracing paper, colour pencils, a pair of scissors, geometry box, gluestick, etc.

Procedure :

1. On a white sheet of paper, draw a parallelogram ABCD.

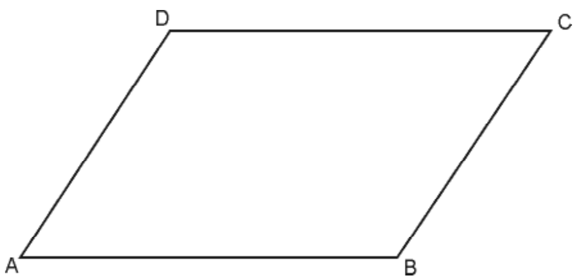


Figure-1

2. Taking the same base AB, draw another parallelogram ABEF, which lies between the parallel lines AB and FC. Shade the three parts using different colours as shown.

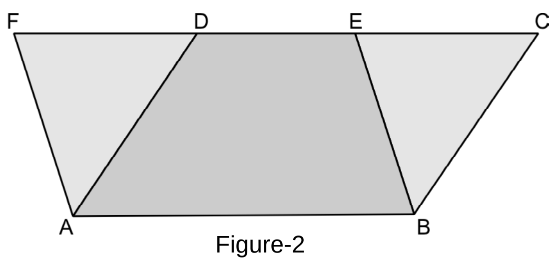


Figure-2

3. On a tracing paper, trace the triangle BCE and cut it out.

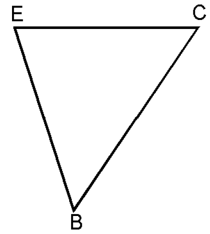


Figure-3

4. Paste the triangular cut out BCE over ΔADF as shown in the figure.

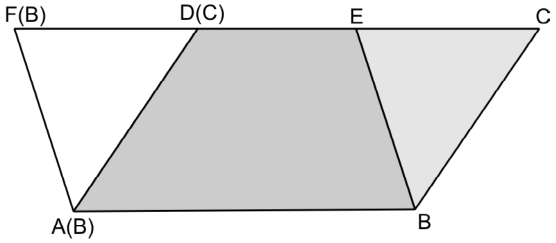


Figure-4

Observations :

1. In figure 2, parallelogram ABCD and ABEF are on the same base AB and between the same parallels AB and CF.
2. Region ABED is common to both the parallelograms.
3. In figure 4, when the traced copy of ΔBCE is placed over ΔADF, we see that both the figures exactly cover each other.
   So, ΔBCE ≅ ΔADF
4. Now, area of trapezium ABED + area of ΔBCE = area of trapezium ABED + area of ΔADF
   ⇒ area of parallelogram ABCD = area of parallelogram ABEF

Conclusion : From the above activity, it is verified that area of the parallelograms standing on the same base and between the same parallels are equal in area.

Activity-4

Objective : To verify by activity method that the triangles on the same base and between the same parallels are equal in area.

Materials Required : White sheets of paper, tracing paper, colour pencils, a pair of scissors, gluestick, geometry box, etc.

Procedure :

1. On a white sheet of paper, draw two triangles ABC and ABD on the same base AB and between the same parallels AB and DC.

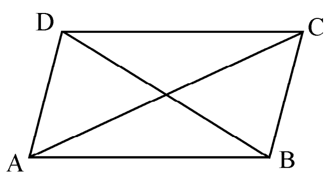


Figure-1

2. Trace the ΔABD on a tracing paper. Cut it out and colour it as shown.

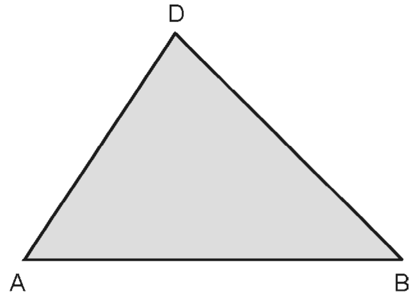


Figure-2

3. Paste the triangular cut out ABD adjacent to ΔABD such that AD and DA coincide as shown in the figure.

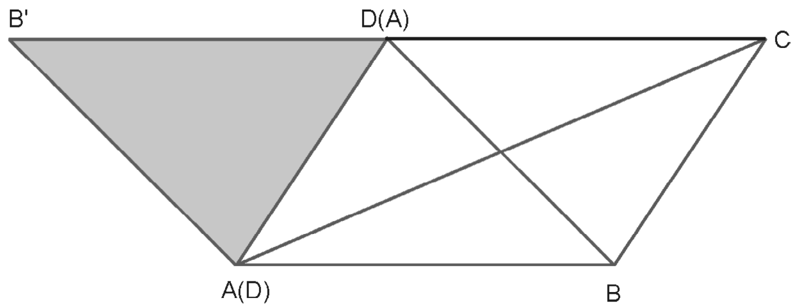


Figure-3

4. Trace the ΔABC on a tracing paper. Cut it out and colour it as shown.

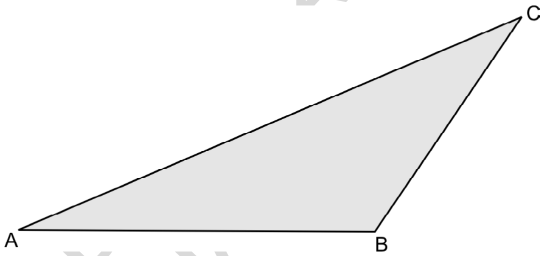


Figure-4

5. Paste the triangular cut out ABC adjacent to ΔABC such that BC and CB coincide as shown in the figure.

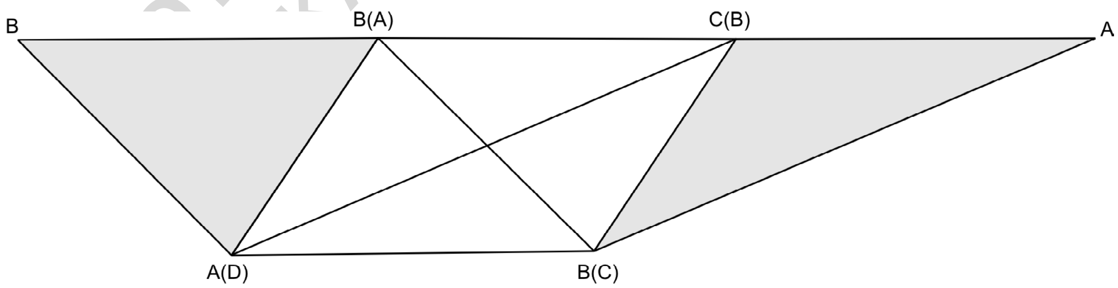


Figure-5

Observations :

1. In figure 1, ΔABC and ΔABD are on the same base AB and between the same parallels AB and DC.
2. In figure 5, ABDB´ is a parallelogram with diagonal AD and ABA´C is a parallelogram with diagonal BC.
3. Parallelograms ABDB´ and ABA´C are on the same base AB and between the same parallels AB and A´B´.

   So, area of parallelogram ABDB′ = area of parallelogram ABA′C

   $\Rightarrow \frac{1}{2}$ area of parallelogram ABDB′ = $\frac{1}{2}$ area of parallelogram ABA′C

   $\Rightarrow$ area of ΔABD = area of ΔABC

Conclusion : From the above activity, it is verified that the triangles on the same base and between the same parallels are equal in area.

ANSWERS

Practice Exercise 9.1A

1. (*d*) **2.** (*d*)

Practice Exercise 9.2A

1. (*b*) **2.** (*b*) **3.** (*c*) **4.** (*d*) **5.** (*b*) **6.** (*b*) **7.** (*c*) **8.** (*d*) **9.** (*b*) **10.** (*a*) **11.** yes **13.** 8.75 cm **14.** No **17.** 12 cm^2

Practice Exercise 9.3A

1. (*a*) **2.** (*a*) **3.** (*c*) **4.** (*c*) **5.** (*b*) **6.** (*c*) **7.** (*b*) **8.** (*c*) **9.** False **11.** 8 cm^2